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Well posed solution of Schwarzschild integral equation in case when the extinction law is known

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In paper [2] we investigated the well posed solution of Schwarzschild integral equation (see Theorem 1 in [2], p. 85)

$$\int_{-\infty}^{+\infty} D(r)\,\varphi(m+5-5\log r - A(r))\,r^2\,dr = A(m), \tag{1}$$

which may be written in the convolution form

$$\int_{-\infty}^{+\infty} \Delta_1(R')\,\varphi(\mu - R')\,dR' = \alpha_1(\mu), \tag{2}$$

where it was assumed, that the extinction $A(r)$ =constans.

Now in this paper, considering the relationship

$$m = M + 5\log\frac{r}{10} + A(r) \tag{3}$$

with help of easy transformations and substitutions

$$y = 5\log\frac{r}{10} + A(r), \tag{4}$$

$$\frac{y - A(r)}{5} = \log\frac{r}{10},$$

$$r = 10\cdot 10^{0.2(y-A(r))} \tag{5}$$

we obtain, that

$$dV = \omega r^2 \, dr = \frac{\omega}{3} (dr^3)$$

and further, that

$$dV = \frac{\omega}{3} 10^3 \frac{d(10^{0.6(y-A(r))})}{dy} \, dy =$$

(6) $$\frac{1}{5 \log e} \frac{\omega}{3} 10^3 10^{0.6(y-A(r))} \left(1 - \frac{dA(r)}{dy}\right) dy.$$

Putting

$$\Omega_y = \frac{1}{5 \log e} \omega 10^3 \Delta_y(y) 10^{0.6(y-A(y))} \left(1 - \frac{dA(y)}{dy}\right)$$

or

$$\Omega_y = \frac{\omega}{3} 10^3 \Delta_y(y) \frac{d}{dy} \left(10^{0.6(y-A(y))}\right)$$

we obtain the equation

$$\frac{1}{5 \log e} \omega 10^3 \int_{-\infty}^{+\infty} \Delta_y(y) 10^{0.6(y-A(y))} \left(1 - \frac{dA(y)}{dy}\right) \varphi(m-y) \, dy = A(m).$$

Designing $\Delta_f = \Delta_y(y) \cdot 10^{-0.6A(y)} \left(1 - \frac{dA(y)}{dy}\right)$ we find that

$$\frac{1}{5 \log e} \omega 10^3 \int_{-\infty}^{+\infty} 10^{0.6y} \Delta_f(y) \, \varphi(m-y) \, dy = A(m)$$

and substituting still $y = m - M$ we obtain, that

$$\frac{1}{5 \log e} \omega 10^3 10^{0.6m} \int_{-\infty}^{+\infty} 10^{-0.6M} \Delta_f(m-M) \, \varphi(M) \, dM = A(m)$$

and further, that

$$\frac{1}{5 \log e} \omega 10^3 \int_{-\infty}^{+\infty} 10^{0.6(m-M)} \Delta_f(m-M) \, \varphi(M) \, dM = A(m).$$

Designing $\beta = \dfrac{1}{5\log e}$ and $\Delta_0(m-M) = 10^{0.6(m-M)}\Delta_f(m-M)$ we obtain the equation in the convolution form

$$\beta\omega 10^3 \int_{-\infty}^{+\infty} \Delta_0(m-M)\,\varphi(M)\,dM = A(m) \tag{7}$$

or finally we obtain, that

$$\beta\omega 10^3 \int_{-\infty}^{+\infty} \Delta_0(M')\,\varphi(m-M')\,dM' = A(m). \tag{8}$$

We can easily verify, that to the last equation we may apply the Theorem 1 from the paper [2]. That means, that in the case when the extinction law is given, the solution of the Schwarzschild integral equation is well posed in some ball, and it means, that the solution of Schwarzschild integral equation in case when the extinction law is know is stable in this ball (cf. [2]).

However we have $\omega r^2\Delta(r) = \Omega_y(y)\dfrac{dy}{dr}$, where $y = 5\log\frac{r}{10} + A(r)$, and therefore we obtain, that

$$\frac{dy}{dr} = \frac{1+\beta r\frac{dA(r)}{dr}}{\beta r}. \tag{9}$$

That means, that we have

$$\Delta(r) = \frac{1+\beta r\frac{dA(r)}{dr}}{\beta\omega r^3}\,\Omega\left(5\log\frac{r}{10} + A(r)\right), \tag{10}$$

where Ω is the solution of the equation in convolution form

$$\int_{-\infty}^{+\infty} \Omega(y)\,\varphi(m-y)\,dy = A(m). \tag{11}$$

This theory has been used to obtain the density function $D(r)$ of stars from star counts in some sky field. From four fields (in *Aquila*, *Aquila–Sagitta*, *Sagitta* and *Cassiopeia*) selected at the Astronomical Observatory of N. Copernicus University in Toruń about thirty years ago the field in *Cassiopeia* has got rich photometric material. With the aid of this material we made an attempt to calculate space density function of stars $D(r)$. The field of *Cassiopeia* is nearly 18.1 square degrees and is centered at $\alpha_{1950} = 23^h57^m$, $\theta_{1950} = 59.6°$. Hutorowicz (1956) used the plates taken with the 8" Draper astrograph at the Toruń Observatory to obtain the photographic magnitudes m_{pg} of stars; Ampel (1958) obtained the photovisual magnitudes m_{pv}. The catalogue of photographic magnitudes contains 1730 stars and probably is complete up to $13^m.00$. The catalogue of photovisual magnitudes contains 3856 stars—probably complete to $13^m.20$. Gertner (1979) has made the transformations of the photographic magnitudes m_{pg} and of the photovisual magnitudes m_{pv} of these stars to the photoelectric B, V system assuming the transformation formulae determined by Wegner (1978).

From star counts see Table 1 and Fig. 1 we have calculated the absolute frequency function $A(m)$ for the stars contained within a solid angle of one square degree (in this work the apparent magnitude m denotes the visual magnitude V, however the absolute magnitude $M = M_V$). We have obtained

$$A(m) = 120.61e^{-0.520(m-12.5)^2} \quad (12)$$

Table 1

$m=V$	$A(m)$	$m=V$	$A(m)$	$m=V$	$A(m)$	$m=V$	$A(m)$
8	1.60	10	16.52	11.5	49.34	13	63.31
8.5	3.20	10.5	24.25	12	84.97	13.5	5.97
9	5.97	11	36.80	12.5	129.61	14	2.98
9.5	8.78						

The spectral classification of all stars brighter than $m_{pv} = 13^m.20$ (1164 stars) was derived from the plates taken with the aid of 24"/36" Schmidt telescope of the Warner and Swasey Observatory equipped with objective prism 4° and the plates from the Stockholm Observatory taken with the aid of the 16" astrograph (objective prism 4.8°). Two additional plates were taken at the Toruń Observatory with the aid of 24"/36" Schmidt-Cassegrain telescope and the 5° flint (F_2) objective prism.

The interstellar extinction curve was built on the base of the colour excess method. The colour excess $E(B-V)$ has been plotted versus the apparent distance modulus $y = V - M_V$ as the first step in construction of the interstellar extinction curve—see Fig. 2.

The next step was the adoption of the value of total to selective extinction ratio

$$(13)\qquad R = \frac{A_V}{E(B-V)}.$$

Putting $R = 3.96$ (≈ 4.0)—see Wegner (1988) we obtain the extinction A_V as a function of distance

$$(14)\qquad V - M_V = 5\log\frac{r}{10} + A_V.$$

Then the extinction as a function of distance has been calculated as $\frac{dA(r)}{dr}$.

The luminosity function $\varphi(M_V)$ fulfilling the condition

$$(15)\qquad \int_{-\infty}^{+\infty}(M)\,dM = 1$$

was obtained from the sample of 1164 stars, for which we know spectral types and luminosity functions. The derived relative frequency distribution $\varphi(M_V)$ versus M_V is shown in Fig. 3. The least—squares fit to the above distribution gives the formula

$$(16)\qquad \varphi(M_V) = 0\overset{m}{.}18e^{-0.102(M_V-4)^2}.$$

This result is very strongly dependent on data for stars with $M_V < -3$ and $M_V > 5$. This fact influences the density function $D(r)$. Further publications will present the results for the $A(m)$ and $\varphi(M)$ functions concerning stars groups for example $B2 - B3, B8 - A0, A2 - A5$ and so on.

Putting the results (12) and (16) into equation (10) we have received the density function $D(r)$ in the form

$$(17)\qquad D(r) = \frac{1 + \beta r\frac{dA(r)}{dr}}{\beta\omega r^3}187.24e^{-0.120(5\log\frac{r}{10}+A(r))^2}.$$

The run of this function is given in Fig. 4. This figure contains also the results received by Ampel (1959) for several types and the run of the density function $D(r)$ calculated with the aid of the simple formula—see Kuroczkin (1958)

$$(18)\qquad A(m) + 1.01510^{-4}D(\bar{r})\left(r_2^3 - r_1^3\right)$$

where $\bar{r} = \frac{r_1 + r_2}{2}$.

In further publications detailed data of $A(m)$ and $D(r)$ functions for *Aql*, *Aql-Sqe*, *Sqe* and *Cass* fields for several spectral types will be discussed.

Fig. 4.

Abstract.

The four fields have been selected at the Astronomical Observatory of N. Copernicus University in Toruń about 30 years ago in order to investigate the structure of Milky Way. The field in *Cassiopeia* has been observed most efficiently and a lot of photometric data concerning this field is collected. This material has been used to derive the function $D(r)$ representing the surface density of stars. This work contains preliminary results. The detailed results will be discussed in further publications.

Streszczenie.

W tym artykule rozważa się poprawność rozwiązania całkowego Schwarzschilda w przypadku, gdy dane jest prawo ekstynkcji. W pracy pokazano wykresy funkcji $A(m)$, $\varphi(M)$ oraz $\log D(r)$ w systemie B, V otrzymane na bazie fotometrii fotograficznej i klasyfikacji widmowej wykonanej przez Ampela (1959) oraz Wegnera (praca przygotowywana do druku) dla gwiazd w polu *Cassiopeia* ($\alpha_{1950} = 23^h 57^m$, $\theta_{1950} = 59.6°$; 18.1 stopni kwadratowych). W dalszych pracach przedstawione zostaną rezultaty obliczeń dla innych pól wyselekcjonowanych w Obserwatorium Astronomicznym Uniwersytetu M. Kopernika w Toruniu.

O poprawnym rozwiązaniu równania całkowego Schwarzschilda w przypadku, gdy prawo ekstynkcji jest znane.

References.

[1] R. Ampel, *Bull. obs. astr.Toruń*, Vol. II, No 3, 1959.

[2] T. M. Jędryka and W. Wegner, *Problemy Matematyczne*, z. 7, p. 76, 1985.

[3] H. Hutorowicz, *Bull. obs. astr. Toruń*, Vol. I, No 2, 1956.

[4] J. Gertner, *Bull. obs. astr. Toruń*, Vol. VI, No 2, 1979.

[5] N. E. Kuroczkin, *Astronomiczeskij żurnal*, tom XXXV, p. 86, 1958.

[6] J. Musielak, *Przestrzenie funkcji całkowalnych*, Pol. P-ń, 1974.

[7] W. Pogorzelski, *Integral equation and their applications*, Warszawa 1966.

[8] K. Schwarzschild, *Über die Integralgleichung der Stellarstatistik*, Astr. Nachr. 185, pp. 81–88, 1910.

[9] S. L. Sobolev, *Urawnienija matematiczeskoj fiziki*, Gostechizdat, Moskwa 1954.

[10] L. Schwartz, *Theorie des distributions*, Act. Sc. et industr. Nr 1091 (t. I), 1950 and Nr 1122 (t. II), 1951, Paris, Hermann et Cie.

[11] A. N. Tichonov, *O rieszeni niekorrektno postawlennych zadacz i metodie regularizacji*, DAN SSSR 1963, t. 151, No 3, pp. 501–504.

[12] R. J. Trumpler, H. F. Weaver, *Statistical Astronomy*, p. 240, Los Angeles 1953.

[13] D. Przeworska-Rolewicz and S. Rolewicz, *Equation in linear spaces* PWN W-wa 1968.

[14] W. Wegner, *Bull. obs. astr. Toruń*, Vol. VI, Nr 1, 1978.

[15] W. Wegner, in press, 1988.

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Figure captions:

Fig. 1. The absolute frequency function $A(m)$ per one square degree versus apparent magnitude $m = V$

Fig. 2. Colour excess $E(B - V)$ versus distance modulus $V - M_V$ in *Cassiopeia* field

Fig. 3. The relative frequency distribution of luminosity function $\varphi(M_V)$ as a function of absolute magnitude M_V

Fig. 4. Density function $\log D(r)$ per $10^3 pc^3$ versus distance r. Circles—our results, crosses—Ample's results, points—results derived from equation (18)