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Conducting "Problem Lectures" by academic teachers and "Control Self-Dependence"

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Of late years, the programmes of teaching mathematics in primary and secondary schools in Poland have been updated rather considerably in the direction of the modernization of the contents being transmitted. Also, a change of the range of knowledge the teacher should possess is connected with this fact. The mathematics which the student learns to know has become, on one hand, more interesting, but on the other hand, a high degree of abstraction has resulted in that many students do not understand the contens conveyed to them or know individual questions only superficially, not realizing the deep contents hidden in the facts transmitted to them and not comprehending the possibilities created by the knowledge they acquire, either. It seems that the main reason for such a state of affairs is the lack of balance between the updating of the programme and the modernization of the teaching programme and the didactic means that are at the university teachers disposal.

Many specialists dealing with questions of the didactics of mathematics in a higher school think that the modernization of the methods used is connected with the necessity of going away from traditional forms or ways of transmitting the contens. What is especially strongly criticized in this case is the lecture. It is proposed to replace it by

other forms like, for instance, a seminar, a self-dependent work of students with a book or a suitably programmed computer. However, the inquiries carried out showed that both the students and the academic teachers pronounce, in a univocal way, for the keeping up of the lecture as the basic form of the transmission of knowledge in a university.

Here are the results included in the paper *"Lecture in the higher school"* (Warsaw 1968, State Scientific Publishers, p. 43) by K. Kruszewski, connected with the inquiry carried out among 59 lecturers and 455 students:

- *the view that the lecture should be applied on a large scale was taken by 61.9% of students and 80.0% of lecturers;*
- *the opinion that the lecture should not be applied in the didactic process in a higher school was held by 2.1% of students, which was not shared by any lecturer.*

In the case of my investigations (carried out among students exclusively), the results are still more convincing: 81.3% of students pronounced for the lecture as a form of transmitting knowledge, dominating in the process of studies.

These results prove that, at present, we are not prepared to give up the lecture as a form of transmitting knowledge and even to reduce its role in an essential way, either. It is beyond any doubt, however, that the traditional lecture (the lecturer writing down definitions, theorems and proofs and the students passively copying the facts given to them) has had its days and does not adhere to the modern didactic process. For it does not make the students active, does not create a positive motivation to learning and does not make easier for them to comprehend the contents being conveyed. The cognitive success of the students depends then, to an immense degree, on the "oratorical talent" of the lecturer, his force of argument, and for the students' part, on the ability to remember facts. However, in this case the greatest defect is the passiveness of listeners.

So, the question may arise whether it is possible to conduct a lecture in a problem way for, say, seventy or a hundred listeners. How to activate such a numerous group of students, how to make them identify themselves with what the lecturer discusses, how to make mathematics be "their mathematics", and not something strange, difficult and unintelligible? After all, one cannot carry out each particular proof in a

problem way and aim at the "discovering" of each particular definition by the students, either. Time does not allow for this (too much knowledge must be conveyed to the students), the fact that we would have to talk with too numerous a group of students, either. The answer to the above question is included in the following three points being proposals for a modern solution of the lecture.

1. Local activity.

The local activity of students must be replaced by their global activity. This is the most important proposal though it has to be complemented and enriched by remaining ones. Preventing the students from building small fragments of individual theories (e.g. proofs of theorems), we must make them participate in the building of general conception of the lecture. Thus, their activity must be directed so as to point out the whole lines of themes. At the same time, the students themselves ought to indicate the ways in which we shall build a given theory.

The academic teacher's task is to create a suitable problem situation and analyse the problem aloud, to encourage the students in seeking appropriate lines of themes and to make them believe that they are capable of building such lines.

The realization of the conception of the lecture, proposed by the students, causes the growth of their interest in the given question. In this way, it is also possible to activate the group of students whose proposals have been rejected. Their natural "juvenile rebellion" can easily be transformed into an attempt of showing us (i.e. lecturers) that it was them who were right, and that their conception was valid. How precious this rebellion is! In order to prove their arguments, they will have to acquaint themselves with "our theory" and build theirs. There is no need to persuade anybody that such situation is very profitable, no matter whose arguments will eventually prevail.

But the considerations presented above may give rise to the questions: what shall we do if the students offer many (not always reasonable) proposals? The answer is contained in the second point of our proposals for the updating of the lecture.

2. Conversation.

The lecture should be conducted in the form of a conversation. The lecturer ought to carry on a dialogue, and since a discission with too numerous a group of students is impossible, therefore he should carry on dialogue with himself—"think aloud" in a way. He must argue pro and con, justify the given assertion aloud and present its weak points. Of course, there is no need to do this in a factitious, theatrical manner. The lecturer should rather present some virtues and faults of each particular conception. It would also be advisable in the course of realizing the conception thus "discussed" to point out what the realization of the material would look like if we accept another solution. Thanks to the conversatory lecture, we can accept the most advantageous, in our opinion, of many proposals of the students; we can also make the students witness in a way, the creation of separate elements of the given theory. In this case, the lecturer, through the loud formulation of doubts and the deliberation over the choice of the most advantageous course of the proof or the loud "discovery" of the definition, creates mathematics in the sight of the students, and does not confine himself only to giving an account of something that was already proved and discovered before.

The successive doubt may be connected with the question: will the students always find the way, are they able to "discover" the most appropriate lines of themes, is the self-dependence left for them in this field beyond their reach or not? The answer to this question is given by the third point of our proposals.

3. Controlled self-dependence.

Before I discuss this topic, I will present an anecdote. The chief bodyguard of one of the presidents said: Mr President, during a meeting with citizens of our country you may greet anybody you want. But my task is to care that the man you shake hands with be my man.

Our situation is similar—leaving a lot of self-dependence to the students, we must keep watching all the time that it does not pass into chaos.

Control is usually connected with an unpleasant, repressive testing of achievements. No such control is thought of in this case. Leaving the establishment of certain conceptions of the lecture to the students, we must state precisely the aims we are going to attain and control whether

the adoption of the given conception will let those aims be attained. The defining of the aims should not be treated as a limitation of the students self-dependence since they may be formulating by the students themselves as, for instance, a result of earlier considerations presented by the lecturer.

Controlled self-dependence lies also in steering the students thoughts, showing them historical aspects of the question and the doubts that mathematicians had to do with in the past as well as convincing the students of the trends of modern mathematics in order to demand from them later some proposals and plans concerning the creation of the given theory or the establishment of lines of themes.

So, controlled self-dependence is a spontaneous discovering of mathematics by the students prepared for the given discovery and controlled by the lecturer, the colleagues and himself, and finding to what extent his discoveries are compatible with expectations and, thereby, to what extent his suggestions are better than other proposals. Of course, leaving a lot of freedom to the students, one should never neglect their "discoveries", even if they were decidedly abortive. For the control of self-dependence has in view not to reject one's conception, but to modify it so that it should possess the properties allowing one to attain the designed aims.

The solution presented here was repeatedly tested in such a way that, during the lecture (in algebra or topology), some part of it was conveyed in the traditional form, and some other fragment was realized in the manner given above. The investigations were therefore conducted in the natural circumstances, the material having been so chosen that its degree of difficulty for both parts be the same.

Here are the results of investigations, experiments and observations:

- *The degree of mastering the material—decidedly higher in the case of the proposed method than in the case of the traditional methods (in the four-grade scale used in Poland for gauging students, the mean difference was 0.83).*
- *With the new method, the students were more eager to acquaint themselves with questions exceeding the examination requirements (in the case of the traditional method, only 6.2% of students widened their knowledge, while in the case of the modernized method, the percentage went up to 37.1).*

• *When the lectures were conducted by means of the modernized method, the attendance at them increased considerably (roughly, by 15–20%).*

• *The new method, however, causes the lengthening of time for realizing the material (roughly, by 10–20%).*

• *68.2% of students expressed their eagerness to continue the activities connected with questions discussed by means of the new method, while only* 40.1% *want to deepen their knowledge of topics discussed traditionally.*

Limited amount of place does not allow us to discuss all those problems. For instance, examples of realization are omitted. I would like, however, to emphasize that the experiments carried out proved that, in the case of the modernized way of conducting the lecture presented earlier, the activeness of the students increases considerably, as well as their becoming emotionally involved, and this facilitates and accelerates the process of creation of their mathematical maturity.

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