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Decomposable subsets are S-convex

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Let $\langle \Omega, \Sigma, \mu \rangle$ be a measure space, where Σ is a σ-algebra of subsets if Ω and μ is a finite nonatomic measure on Σ. Given a Banach space E with the norm $|\cdot|$, let $L^1 = L^1(\Omega, \Sigma, \mu, E)$ be the Banach space of all Bochner μ-integrable functions $y : \Omega \longrightarrow E$, endowed with the norm

$$\| y \|_1 = \int_\Omega | y(\omega) | \, d\mu(\omega). \tag{1}$$

A subset $K \subset L^1$ is said to be decomposable (cf. [14], [20], [2]–[4], [10], [11], [5]) if $k_1, k_2 \in K$, $A \in \Sigma$ implies $I_A \cdot k_1 + I_{\Omega\setminus A} \cdot k_2 \in K$, where I_B stands here for the indicator (= characteristic function) of a subset $B \in \Sigma$. Decomposable subsets have many applications in the theory of multifunctions, since the set of all integrable selections for a given multifunction $F : \Omega \longrightarrow E$ is decomposable. In the present note we investigate the relationships between decomposability and others kinds of generalized conexity ([12], [13], [6], [15] – [18], [30]), mainly the S-convexity introduced by L. Pasicki (see [21] – [25], [28] – [29]). This allows us to deduce, as a corollaries of existing for S-convexity results, some new and seemingly interesting properties of multifunctions with decomposable values. In the sequel Y $(= L^1)$ will allways denote a separable Lebesque space.

Definition 1 [16] A C-convexity on a set Y is a collection C of subsets of Y such that C is closed under formation of intersections, in particular

$\emptyset \in C$ and $Y \in C$. Associated to a convexity C on Y is a hull-operator h_C defined on $B \subset Y$ as follows

$$(2)\quad P(Y) \ni B \longmapsto h_C(B) \;=\; \bigcap\{K : B \subseteq K \subseteq C\} \subseteq C \subseteq P(Y),$$

where $P(Y) = \{B : B \subseteq Y\}$.
The hull of a finite set is called a polytope. After Hammer [13], a convexity structure C having the property

$$(3)\qquad h_C(B) = \bigcup\{h_C(T) : \; T \subseteq B \;, \; \operatorname{card} T < \aleph_0\}$$

will be called domain finite.

Proposition 1 *The family C of all decomposable subsets of Y constitutes a C-convexity which is domain-finite.*

Proof. Obviously the intersection of any family of decomposable subsets is decomposable. Denote by *dec* the operator (2) defined for the family C of decomposable sets. We give another characterization of this hull-operator. Let Δ_n, $n = 1, 2, \ldots$ be the family of all n-tuples $\langle A_1, A_2, \ldots, A_n\rangle$ of measurable subsets $A_i \in \Sigma$ such that

$$(4)\qquad \mu(\Omega \setminus \bigcup_{i=1}^{n} A_i) = 0$$

and

$$(5)\qquad \mu(A_i \cap A_m) = 0 \quad \text{if} \quad m \in \{1, 2, \ldots, n\} \setminus \{i\}.$$

Next define for $n = 1, 2, \ldots$ and for $B \subseteq Y$

$$(6)\quad h_n(B) \;=\; \{I_{A_1}y_1 + I_{A_2}y_2 + \ldots + I_{A_n}y_n \in Y : \; y_1, \ldots, y_n \in B, \\ \langle A_1, A_2, \ldots, A_n\rangle \in \Delta_n\},$$

and, assuming $h_0(B) = B$, let us put

$$(7)\qquad h(B) = \bigcup_{n=0}^{\infty} h_n(B).$$

Observe, that for any subset $B \subseteq Y$ we have $decB = h(B)$. In fact, if $k_1 \in h_n(B)$ and $k_2 \in h_m(B)$ then clearly $I_A k_1 + I_{\Omega\setminus A} k_2$ belongs to $h_{n+m}(B)$ which in turn is contained in $h(B)$. Thus $h(B)$ is decomposable and bearing in mind that $B \subseteq h(B)$, according to (2), $dec(B) \subseteq h(B)$. On the other hand, if K is any decomposable subset containing B, then clearly $h_n(B) \subseteq K$ for all $n = 1, 2, \ldots$.

In order to prove (3) observe that

(8) $$h_n(B) = \bigcup\{\, dec\{y_1, y_2, \cdots, y_n\} \;:\; \langle y_1, y_2, \cdots, y_n\rangle \in Y^n\}.$$

That ends the proof.

Remark 1 The closure operator (2) enjoys certain properties identical to those of the closure operator in topology, among which are:

(9) $$B \subseteq dec\, B, \qquad B \in P(Y),$$

(10) $$B_1 \subseteq B_2 \quad\Longrightarrow\quad dec\, B_1 \subseteq dec\, B_2$$

(11) $$dec(dec\, B) = dec\, B$$

(12) $$B \in C \quad\Longleftrightarrow\quad dec\, B = B.$$

Definition 2 [31] If, in addition to convexity C, the set Y also carries a topology, then $\langle Y, C\rangle$ is called a topological convex structure (cf. also [16], [17], [30]) provided all polytopes are closed. In a topological convex structure we may define a closed-convex hull operator

$$h_C : P(Y) \longrightarrow P(Y)$$

by formula

(13) $$h^*_C(B) = \bigcap\{D : \quad B \subseteq D = clD \in C\},$$

where $cl : P(Y) \longrightarrow P(Y)$ is the closure operator in the topological space Y. Note that in general $h^*_C(B)$ differs from $cl h_C(B)$. The equality $h_c(B) = cl\, h_C(B)$ holds if our topological convex structure $\langle Y, C, cl\rangle$ is closure stable, it means the closure of a C-convex set is C-convex again.

Proposition 2 *The family C of all decomposable subsets of Y constitutes a topological convex structure which is closure-stable.*

Proof. Observe that the limit of a sequence of terms of the form $I_{A_1}y_1 + I_{A_2}y_2 + \ldots + I_{A_n}y_n$ with fixed $y_1, y_2, \ldots, y_n$ and $\langle A_1, A_2, \ldots, A_n\rangle$ runing over Δ_n must be of the same form, so that all polytopes are closed. It is also easily checked, by passing to the limit, that the closure of any decomposable set must be decomposable.

Following Pasicki ([21] - [26]) a set Y is S-linear if there is a mapping $S : Y \times [0,1] \times Y \longrightarrow Y$ such that $S(a,0,b) = b$ and $S(a,1,b) = a$ for all $a, b \in Y$. Note that the pair $\langle Y, S\rangle$ is a convex prestructure in the sense of Gudder, Schroeck [12]. For any subset B of S-linear set Y define

$$(14)\quad coS(B) = \bigcap\{D \subseteq Y : \quad B \subseteq S*(B \times [0,1] \times D) \subseteq D\},$$

where

$$S*(B \times [0,1] \times D) = \bigcup_{a\in B}\ \bigcup_{0\leq t\leq 1}\ \bigcup_{b\in D}\{S(a,t,b)\} \subseteq Y.$$

Observe that $coS : P(Y) \longrightarrow P(Y)$ is a preconvex hull operator on Y that means that the following two conditions, analogous to (9) and (10) are satisfied

$$(15)\qquad B \subseteq coSB \quad \text{for any} \quad B \subseteq Y,$$

$$(16)\quad B_1 \subseteq B_2 \implies coS\, B_1 \subseteq coS\, B_2 \ \text{ for any } \ B_1, B_2 \subseteq Y.$$

Therefore the family

$$(17)\qquad C_S = \{B \subseteq Y : \quad B = coS\, B\}$$

is a C-convexity on Y. There exists an example showing that coS may fail to be a convex hull operator, namely $coS\ coS \neq coS$ in general (see [28]). But if we define h_{C_S} as in (2) then there exists an (possibly transfinite) iteration $coS \circ coS \circ coS \circ \ldots coS \circ \ldots$ giving $h_{C_S} = (coS)^\alpha$. Evidently $coS \leq h_{C_S}$ on $P(Y)$. In case when the space $\langle Y, S\rangle$ is endowed with a topological structure, it is then to impose certain continuity-like conditions on S. An S-linear topological space is S-contractible if $S(a,\cdot,\cdot):[0,1]\times Y\longrightarrow Y$ is a homotopy joining the identity $S(a,0,\cdot)$ with a constant map. In the special case when S is continuous on $Y\times[0,1]\times Y$ as a function of 3 variables, the above notion coincides with the notion of equiconnected space, as defined in [6] - [9], [15]. In this case S is called an equiconnecting function.

Proposition 3 *There exists an equiconnecting function*

$$S: Y\times[0,1]\times Y\longrightarrow Y$$

for which each decomposable subset of Y is S-convex.

Proof. First define a multifunction $H: Y\times Y\longrightarrow Y$ by formula

$$H(y_1,y_2)=dec(\{y_1,y_2\})\in C. \tag{18}$$

We shall prove that H is lower semicontinuous on the space $Y\times Y$. This means that $y^0\in H(y_1^0,y_2^0)$ and U open containing y^0 imply that there is an open set G in $Y\times Y$ such that whenever $(y_1,y_2)\in G$ then $H(y_1,y_2)\cap U\neq\emptyset$. Let U be an open set in Y. We may suppose without loss of generality that U is a ball $U=B(y,r)\subseteq Y$. If $H(y_1^0,y_2^0)\cap U\neq\emptyset$, then there exists a measurable subset $A\in\Sigma$ such that

$$\| I_A\, y_1^0+I_{\Omega\setminus A}\, y_2^0-y\|_1<r. \tag{19}$$

Thus there is a positive number r_1 for which

$$\int_\Omega | I_A y_1^0(\omega)+I_{\Omega\setminus A}y_2^0(\omega)-y(\omega)|\,d\mu(\omega)=r-r_1. \tag{20}$$

For any $y_i\in B(y_i^0,\frac{r_1}{3})$, $i\in\{1,2\}$, the following estimate holds

$$\begin{aligned}
\| I_Ay_1+I_{\Omega\setminus A}y_2-y\|_1 \;&\le\; \| I_Ay_1-I_Ay_1^0\|_1+ \\
&+\| I_Ay_1^0-I_Ay-I_{\Omega\setminus A}y+I_{\Omega\setminus A}y_2^0\|_1+\| I_{\Omega\setminus A}y_2^0-I_{\Omega\setminus A}y_2\|_1\le \\
&\le\| y_1-y_1^0\|_1+\| I_Ay_1^0+I_{\Omega\setminus A}y_2^0-y\|_1+\| y_2^0-y_2\|_1\le \\
&\le\frac{r_1}{3}+(r-r_1)+\frac{r_1}{3}=r-\frac{r_1}{3}<r.
\end{aligned} \tag{21}$$

Therefore $H(y_1, y_2) \cap U \neq \emptyset$ for any (y_1, y_2) belonging to the open neighbourhood $G = B(y_1^0, \frac{r_1}{3}) \times B(y_2^0, \frac{r_1}{3})$ of the arbitrarily chosen point $(y_1^0, y_2^0) \in Y \times Y$. Since (y_1^0, y_2^0) was arbitrary, we infer that H is lower semicontinuous on the entire space $Y \times Y$. Next define a multifunction $P : Y \times [0,1] \times Y \longrightarrow Y$ by formula:

$$(22) \qquad P(y_1, t, y_2) = \begin{cases} H(y_1, y_2) & \text{for } \ 0 < t < 1 \\ \{y_{2-t}\} & \text{for } \ t \in \{0, 1\} \end{cases}$$

By the lower semicontinuity of H we infer, by a routine manner, the lower semicontinuity of P on $Y \times [0,1] \times Y$. Since the domain $Y \times [0,1] \times Y$ is separable (as a product of separable spaces) and since H has closed, decomposable values, as it is shown in Proposition 5, we may apply Pasicki's selection theorem ([24], th.3 p. 73) to obtain a continuous selection

$$S : Y \times [0,1] \times Y \longrightarrow Y$$

for multifunction P. This is the required equiconnecting function since, by (22), $S(y_1, 0, y_2) = y_2$ and $S(y_1, 1, y_2) = y_1$. Let $K \subseteq Y$ be any decomposable subset. Obviously $K \subseteq S * (K \times [0,1] \times K)$. On the other hand $S(y_1, t, y_2) \in P(y_1, t, y_2) \subseteq dec\{y_1, y_2\} \subseteq K$ whenever $y_1, y_2 \in K = dec\, K$. Thus $S * (K \times [0,1] \times K) \subseteq K$ and we have finally

$$(23) \qquad K = dec\, K = coS(K).$$

That ends the proof. For a nonempty subset $K \subseteq Y$ and for a positive number r let us write

$$(24) \quad B(K, r) = \{y \in Y : \ d(y, K) < r\} = \bigcup\{B(K, r) : \ k \in K\},$$

where $d(x, K)$ is the distance function induced by the norm in Y.

Definition 3 ([24], df. 5 on p.67) A metric space Y is said to be uniformly of Pasicki's type II for balls if it is S-contractible for an S satisfying the following condition: for any $\varepsilon > 0$ there is $\delta > 0$ such that for any subset $K \subseteq Y$ the following inclusion holds

$$(25) \quad S * (B(K, \delta) \times [0,1] \times B(K, \delta)) \subseteq B(*(K \times [0,1] \times K), \varepsilon).$$

Proposition 4 *The Lebesque space $Y = L_1$ endowed with the equiconnecting function S from Proposition 3 is uniformly of Pasicki's type II for balls.*

Proof. Take a positive real number $r > 0$ and two arbitrary points y_1, y_2 belonging to $B(K, r)$. There exist $k_i \in K$, $i \in \{1, 2\}$, such that $y_i \in B(k_i, r)$. We have

$$(26) \qquad \| I_A y_1 + I_{\Omega \setminus A} y_2 - (I_A k_1 + I_{\Omega \setminus A} k_2) \|_1 =$$

$$\| I_A(y_1 - k_1) + I_{\Omega \setminus A}(y_2 - k_2) \|_1 \leq \| y_1 - k_1 \|_1 + \| (y_2 - k_2) \|_1 < 2r,$$

for each $A \in \Sigma$. Since, by (22) and (18),

$$(27) \qquad S(y_1, t, y_2) = I_A y_1 + I_{\Omega \setminus A} y_2$$

for an adequate $A \in \Sigma$ and since $(y_1, t, y_2) \in B(K, r) \times [0, 1] \times B(K, r)$ was arbitrary, we have in fact that

$$(28) \qquad S(y_1, t, y_2) = B(I_A k_1 + I_{\Omega \setminus A} k_2, 2r).$$

Obviously $I_A k_1 + I_{\Omega \setminus A} k_2 \in S(K \times [0, 1] \times K)$. Thus

$$(29) \quad S * (B(K, r) \times [0, 1] \times B(K, r)) \subseteq B(*(K \times [0, 1] \times K), 2r).$$

Taking $\delta = \frac{\varepsilon}{2}$ we obtain the desired inclusion (25) achieving the proof.

Remark 2 A metric space Y is called uniformly of Pasicki's type 0 for balls (see [24], [25], [26]) if it is S-contractible for an S satisfying the following condition: for any $\varepsilon > 0$ there is a $\delta > 0$ such that for any subset $K \subseteq Y$ we have

$$(30) \qquad coS\ B(K, \delta) \subseteq B(coS\ K, \varepsilon).$$

Obviously there is no S-contraction S on $Y = L^1$ with exactly decomposable sets being S-convex and inverting Y into S-contractible space of type 0 uniformly for balls. In fact $coS\ B(K, \delta) = dec\ B(K, \delta)$ must be unbounded for any nonempty $K \subseteq Y$ as it may be observed by constructing easy examples.

Definition 4 [19] A collection K of nonvoid, closed subsets of a topological space Y is said to be equi-locally connected if for any point $y \in \bigcup\{B : \quad B \in K\}$ and for any open neighbourhood $U \subseteq Y$ of this point y there may be chosen a neighbourhood $V \subseteq Y$ of y with the property that any two points joint by a connected subset of $U \cap B$.

Proposition 5 *The family of all nonempty, closed, decomposable subsets of Lebesque space $Y = L_1$ is equi-locally connected in the Nepomnyashchij sense reminded above.*

Proof. An arc $\mathrm{L} = \{S(y_1, t, y_2) : \quad 0 \leq t \leq 1\}$ is connected and contained in $U \supseteq B(y, r)$ whenever y_1, y_2 belong to $V = B(y, \frac{r}{2})$. Obviously $\mathrm{L} \subseteq B = dec\, B$ if $\{y_1, y_2\} \subseteq B$, achieving the proof.
Let us recall that a multiselection for a multifunction $F : X \longrightarrow Y$ is a second multifunction $G : X \longrightarrow Y$ with the property that $G(x) \subseteq F(x)$ for all $x \in X$. If moreover, $G(x) = \{f(x)\}$, where f is single-valued, then f is called a selection for F. Since each decomposable subset is (e.g. by Proposition 2) arcwise connected, we may apply, in view of Proposition 5, a result of G.M. Nepomnyashchij ([19], th. 1.1) to decomposable-valued multifunctions, obtaining the following theorem on extensions of continuous multiselections

Proposition 6 *Let X be paracompact topological space, $Y = L^1$ the Lebesque space and $F : X \longrightarrow Y$ a lower semicontinuos multifunction with closed, decomposable values. Let A be a closed subset of X and suppose we have a continuous multiselection $G_A : A \longrightarrow Y$ with compact values. Then there exists a continuous multifunction $G : X \longrightarrow Y$ with compact values such that*

$$G(a) = G_A(a) \subseteq F(a) \quad \textit{for all} \quad a \in A = cl\, A, \tag{31}$$

$$G(x) \subseteq F(x) \quad \textit{for all} \quad x \in X. \tag{32}$$

If moreover, all values of G_A are connected, then G may be chosen also with connected values. The continuity of G, G_A is considered, the hyperspace of closed, decomposable subsets being equiped with the exponential Vietoris topology.

Proposition 4 permits us to apply a fixed-point theory for functions and multifunctions with values in S-contractible spaces of type II, developed in [23], [25], [26] especially for multifunctions defined on finite-dimensional paracompact spaces. We include here the only sample of this kind.

Proposition 7 *Let $F : D \longrightarrow Y$ be a compact multifunction defined on a compact subset D of Y and having closed, decomposable values. Then there exists $d \in D$ such that $d \in F(d)$ provided*

$$F(D) = \bigcup\{F(x) : \ x \in D\}$$

is finite dimensional in the sense of cover dimension.

By using Proposition 4 we may also obtain a selection theorem without any assumption on the metrizability of the domain space, in contrast to the results from [2], [3], [10].

Proposition 8 *Let X be a finite dimensional paracompact space. Any lower semicontinuous multifunction $F : X \longrightarrow Y$ with closed, decomposable values admits a continuous selection.*

Proof This follows directly from theorem 2 on p. 67 in [24] due to our Proposition 4.

Definition 5 [19]. Let Y be a topological space, $K = \{B_j : \ j \in J\}$ any family of nonempty closed subsets of Y and

(33) $$C[K] = \bigcup\{C(B_j) : \ j \in J\},$$

where $C(B_j)$ denote the family of all connected and compact subsets of B_j (that means subcontinuous map $l : C[K] \longrightarrow Y$ fulfilling two following axioms

(34) $$l(\{h\}) = h \quad \text{for each singleton} \quad \{h\} \in K$$

(35) $$l(H) \in B_j \quad \text{for each} \quad H \in C(B_j) \ \textit{and} \ B_j \in K,$$

is called a K-preserving retraction.

In the presence of such a retraction Proposition 7 remains true in case of single-valued G and G_A. My conjecture is that the family of all closed decomposable subsets of a separable Lebesgue space L^1 admits a retraction preserving this family in the sense of Definition 5. Since the family H of all compact, connected subsets of $Y = L^1$ endowed with Vietoris finite topology is a metrizable, separable space, it suffices to prove the lower semicontinuity of the following multifunction

$$H \ni B \longmapsto P(B) = cl\, dec(B) \in K \tag{36}$$

and then a selection theorem of A. Bressan and G. Colombo [3] can be applied in order to obtain (33) and (34). That program seems to be difficult. In connection with a result of [27] establishing the structure of the set of fixed points of a multivalued contraction with convex values, we include here the following corollary from [4]

Proposition 9 *Let $F : Y \longrightarrow Y$ be a contractive multifunction with closed, decomposable values, i.e.*

$$d_H(F(u), F(v)) \leq k \parallel u - v \parallel_1, \tag{37}$$

for some Lipschitz constant $k < 1$, all u and v belonging to Y, while d_H denotes the Hausdorff generalized distance

$$d_H(A, B) = \max\{\sup_{a\in A} \inf_{b\in B} \parallel a - b \parallel, \ \sup_{b\in B} \inf_{a\in A} \parallel b - a \parallel \}. \tag{38}$$

Then the set of fixed points of F

$$Fix\, F = \{u \in Y : \ u \in F(u)\} \tag{39}$$

is an absolute retract for separable metric spaces.

It remains an open problem whether the set (39) is an absolute extensor for paracompact spaces, since the separability of Y is essential in the proofs of the results of [3], [4]. Note that boundedness assumed in Theorem 1 in [4] is unessential and that under the assumption that conjecture stated before is true, one can easily obtain a positive answer to this problem. These possibilities will be investigated later.

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