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THE APPROXIMATE CONTINUITY OF L_p SMOOTH FUNCTIONS

A real valued function f defined on the real line $\mathbb{R}$ is said to be smooth at a point $x \in \mathbb{R}$ if

$$(*) \qquad \lim_{t\to 0} \frac{\Delta^2 f(x,t)}{t} = 0$$

where $\Delta^2 f(x,t) = f(x+h) + f(x-h) - 2f(x)$. If, in place of the usual limit in $(*)$, we use the approximate limit, then f is said to be approximately smooth at the point $x \in \mathbb{R}$. Similarly, a measurable function f is said to be L_p $(1 \le p < \infty)$ smooth at $x \in \mathbb{R}$ if $(*)$ holds with the limit taken in the L_p sense. The function f is called smooth or approximately smooth or L_p smooth if it is so at each $x \in \mathbb{R}$. The continuity properties of the associated classes of smooth functions have been studied quite extensively and many of these investigations have focused on identifying the set of those points at which a given function is discontinuous. In specific, Neugebauer showed that if f is measurable and smooth, then $\mathbb{R}-C(f)$ is a nowhere dense countable set [$C(f)$= the continuity points of f] . Subsequently, Evans and Larson showed that for measurable smooth functions, $\mathbb{R}-C(f)$ is characterized as clairseme (or scattered) . In each of the approximately and L_p smooth cases. Neugebauer showed that $\mathbb{R}-C(f)$ can have large measure but that for approximately smooth f, $\mathbb{R}-AC(f)$ has measure zero and for L_p smooth f, $\mathbb{R}-L_pC(f)$ has measure zero. Here, $AC(f)$ denotes the points of approximate continuity of f and $L_pC(f)$ denotes the L_p continuity points of f. As Neugebauer mentions, a natural question is whether the nowhere dense and measure zero set $\mathbb{R}-L_p C(f)$ must be countable for an L_p smooth function f. An associated question is whether the set $\mathbb{R}-AC(f)$ must be countable for an approximately smooth function f. In this lecture, the orator presents a general construction technique whioh shows that in either case the answer is negative. In specific, an appropriately (L_p or approximately) smooth function is constructed such that $\mathbb{R}-AC(f)$ is uncountable and as $L_pC(f) \subset AC(f)$ the result(s) follows.

REFERENCES

[1] Evans M.J. and Humke P.D., A pathological approximately smooth function. Acta Math. Acad. Sci. Hungar to appear

[2] Evans M.J. and Humke P.D., L_p smoothness and approximate continuity, Proc. Amer. Math. Soc.(to appear)

[3] Evans M.J. and Larson L., The continuity of symmetric and smooth functions, Acta Math. Acad. Sci. Hung.(to appear)

[4] Neugebauer C.J. Symmetric, continuous and smooth functions, Duke Math. J. 31 (1964), 23-32

[5] Neugebauer C.J. Smoothness and differentiability in L_p, Studia Math. 25 (1964), 81-91

[6] O'Malley R.J., Baire 1. Darboux functions, Proc. Amer. Math. Soc., 60 (1976) 187-192