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CONCERNING CONTINUOUS SELECTORS FOR MULTIFUNCTIONS DEFINED ON PRODUCT SPACES

Let X be a topological space and (Y, d) a metric space. A multifunction $F: X \to Y$ is called almost lower semicontinuous (cf. [4]) if for each $x_0 \in X$ and each $\varepsilon > 0$ there is a neighbourhood U of x_0 such that:

(1) $\bigcap \{K(F(x), \varepsilon) : x \in U\} \neq \emptyset$ where

(2) $K(F(x), \varepsilon) := \{y \in Y : \operatorname{dist}(y, F(x)) < \varepsilon\} =$
$= \{y \in Y : \inf \{d(y,a) : a \in F(x)\} < \varepsilon\}$.

A multifunction is called lower semicontinuous (shortly lsc) ([9]) if

(3) $F^{-}(U) := \{x \in X : F(x) \cap U \neq \emptyset\}$

is open in X for every open U in Y. Obviously each lower semicontinuous multifunction is almost lower semicontinuous, but not conversely in general. In accordance with a recent work of de Blasi and Myjak [2] a multifunction $F: X \to Y$ is called weakly lower semicontinuous at point $x_0 \in X$ if for each $\varepsilon > 0$ and each open neighbourhood V of x_0 there exist an open neighbourhood U of x_0, $U \subset V$ and a point $x_1 \in U$ such that:

(4) $F(x_1) \subset K(F(x), \varepsilon)$ whenever $x \in U$.

If F is weakly lower semicontinuous at each point $x \in X$, then

F is simply called weakly lower semicontinuous. There are lsc multifunctions without being weakly lsc and vice versa. Each weakly lsc multifunction is almost lsc (see [2], lemma 1 in connection with [4]). For the related notion of nearly lower semicontinuity the reader is refered to [17].

If F is single - valued, say $F(x) = \{f(x)\}$ then almost lower semicontinuity (and thus both lsc and weakly lsc) all reduce to ordinary continuity of $f: X \to Y$. A multifunction $F: X \to Y$ called compact, if the image

(5) $F(x) := \bigcup \{F(x) : x \in X\}$

is relatively compact in X. Paper [2] contains the following selection theorem:

THEOREM 1 ([2]). Let X be a paracompact topological space, Y a Banach space and $F : X \to Y$ a weakly lower semicontinuous multifunction with compact and convex values. Assume that F is a compact in the above sense [5]. Then F has a continuous selector, i.e. a function $f: X \to Y$ such that $f(x) \in F(x)$ for all $x \in X$.

REMARK 1: An inspection of the proof of theorem 1 shows that the space X may be assumed to be K-paracompact only, where K= weight of X.

The aim of the presented paper is to show how the methods developed in [15] can be used to combining the above theorem 1 with famous Michael's selection theorem [10] in order to obtaining a new selection theorem in the spirit of theorem 4.4 from [15].

Let us recall, that the multifunction $F: X \to Y$ is $C_0(X, Y)$-

-stable (cf. [15], def. 1.1) if the following two conditions are satisfied:

(i) $\{f \in Y^X : f(x) \in F(x) \text{ for all } x \in X\} \cap C_0(X, Y) \neq \emptyset$

(ii) for every $r, r_1 \in R^+$ and every $f \in C_0(X, Y)$ such that $F(x) \cap K(f(x), r) \neq \emptyset$ for all $x \in X$, there exists a function $g \in C_0(X, Y)$ such that $g(x) \in F(x) \cap K(f(x), r + r_1)$ for all $x \in X$. Here the sign $C_0(X, Y)$ denote the Banach space of all continuous, bounded maps $f: X \to Y$ equiped with the uniform norm:

(6) $|||f||| := \sup\{\|f(x)\|_Y : x \in X\}$.

LEMMA 1. Let X be a paracompact topological space, Y a Banach space and $F: X \Rightarrow Y$ a weakly lower semicontinuous, compact multifunction with convex and compact values. Let $f: X \to Y$ be a continuous function such that $F(x) \cap \bar{K}(f(x), r) \neq \emptyset$ for each $x \in X$. Then for each $\varepsilon > 0$ there exists a number $\sigma = \sigma(\varepsilon) > 0$ such that

(7) $K(F(x), \sigma) \cap K(G(x), \sigma) \subset K(F(x) \cap G(x), \varepsilon)$

for each $x \in X$, where

(8) $G(x) := \bar{K}(f(x), r + r_1)$ for any fixed positive constant r_1. The sign $\bar{K}(f(x), r)$ denote the closed ball centered at $f(x)$ and with radius r.

PROOF: Suppose, by a way of a contradiction, that there exists an $\varepsilon > 0$ such that for each $\sigma_n := 1/n$, $n \in N$ there is an $x_n \in X$ for which it is possible to construct a sequence (w_n), $w_n \in Y$ satisfying:

(9) $w_n \in K(F(x_n), \sigma_n) \cap K(G(x_n), \sigma_n)$ and moreover

(10) $\operatorname{dist}(w_n, F(x_n) \cap G(x_n)) \geq \varepsilon$ for all $n = 1, 2, \ldots$.

Let us consider a sequence (v_n) of vectors of the space Y such that:

(11) $v_n \in F(x_n) \cap K(f(x_n), r)$ $n \in N$.

Since all the sets $F(x_n)$ are non-void, compact and convex and contained in the compact set cl co $F(X)$, thus the sequences $(F(x_n))$ and $(f(x_n))$ have convergent subsequences. Without any loss of generality we can assume, that $F(x_n)$ tends to A in the Hausdorff metric:

(12) $$h(A,B) := \max\left(\inf_{a\in A}\ \sup_{b\in B}\ d(a,b),\ \sup_{b\in B}\ \inf_{a\in A}\ d(b,a)\right)$$

For, let us recall that the hyperspace of closed and convex subsets of the compact metric space $F(X)$ is a complete metric space with respect to the above metric h(cf. [3]). We cau also assume, that $f(x_n) \to p \in$ cl (oo $F(X)$). From the above mentioned completeness of the hyperspace of simultaneously compact and convex subsets of $F(X)$ we infer that our set A is not only compact, but also convex. Analogously we may without loss of generality assume that $v_n \to v$ and $w_n \to w$ (if n tends to infinity), in such a manner, that

(13) $v \in \bar{K}(p, r)$, $w \in A \cap \bar{K}(p, r + r_1)$ (cf. lemma 2 below)

Let us consider the closed segment $[v, w]$. It is obviously contained in the intersection $A \cap \bar{K}(p, r + r_1)$ and the bound $\partial \bar{K}(p, r+r_1)$ contains no more than one of the ends of that segment (by lemma 2)

Take $\tilde{w} \in [v, w) := [v, w] \setminus \{w\}$ for whioh $\|\tilde{w} - w\| < \varepsilon/3$. We have $\tilde{w} \in A \cap K(p, r+r_1)$. Now let $0 < \eta < \varepsilon/3$ satisfy the condition

(14) $K(\tilde{w}, 2\eta) \subset K(p, r+r_1)$.

Bearing in mind that $f(x_n) \to p$, $n \to +\infty$ we deduce the existence of an $n_0 \in N$ such that for $n \geq n_0$ the inclusion $K(\tilde{w}, \eta) \subset G(x_n)$ holds. On the other hand $\tilde{w} \in A$ and $F(x_n) \to A$ for $n \to +\infty$. From this it follows that there exists an $n_1 \geq n_0$ such that for every $n \geq n_1$ the intersection $F(x_n) \cap K(\tilde{w}, \eta)$ is nonempty. Take an arbitrary point w_n' belonging to this intersection $F(x_n) \cap K(\tilde{w}, \eta)$. Obviously $w_n' \in F(x_n) \cap G(x_n)$. For $n > n_1$ sufficiently large, so that the inequality $\| w_n - w \| < \varepsilon / 3$ holds, we have the following estimates:

(15) $$\| w_n - w_n' \| \leq \| w_n - w \| + \| w - \tilde{w} \| + \| \tilde{w} - w_n' \| < \varepsilon / 3 + \varepsilon/3 + \eta < \varepsilon .$$

Thus $\operatorname{dist} (w_n, F(x_n) \cap G(x_n)) \leq \| w_n - w_n' \| < \varepsilon$. But this is in contradiction with (10) so that the proof of lemma 1 is completed.

LEMMA 2. Let F_1, F_2 be two nonempty bounded closed subsets of the Banach space Y and let $r_1, r_2 \geq 0$ be given constants. If $h(F_1, F_2) \leq \varepsilon_1$ and $|r_1 - r_2| \leq \varepsilon_2$, then

(16) $$h(\bar{K}(F_1, r_1), \bar{K}(F_2, r_2)) < \varepsilon_1 + \varepsilon_2 ,$$

where h is given by the formula [12].

PROOF: We need only to show that $\operatorname{dist}(y, \bar{K}(F_2, r_2)) < \varepsilon_1 + \varepsilon_2$ for every y belonging to $\bar{K}(F_1, r_1)$ because of the symmetry. Given any $y \in \bar{K}(F_1, r_1)$ and an arbitrary positive number $\eta > 0$. there exists a point $y_1 \in F_1$ with $\| y - y_1 \| \leq r_1 + \eta/2$, a point $y_2 \in F_2$ with $\| y_1 - y_2 \| \leq \varepsilon_1 + \eta/2$ and a point y_3

belonging to the segment $co\{y, y_2\}$ with $\|y_3 - y_2\| = \min(r_2, \|y - y_2\|)$. Clearly $y_3 \in K(F_2, r_2)$. If $\|y - y_2\| \le r_2$ then $y_3 = y$ and $\|y - y_3\| = 0$. On the contrary if $\|y - y_2\| > r_2$ then $\|y_3 - y_2\| = r_2$ and :

(17) $\|y - y_2\| \le \|y - y_1\| + \|y_1 - y_2\| < r_1 + \eta/2 + \varepsilon_1 + \eta/2 =$
$= r_1 + \varepsilon_1 + \eta$

from here we infer that

(18) $\|y - y_3\| = \|y - y_2\| - \|y_3 - y_2\| = r_1 + \varepsilon_1 + \eta - r_2 \le$
$\le \varepsilon_2 + \varepsilon_1 + \eta$.

Thus in either case $\text{dist}\,(y, K(F_2, r_2)) \le \varepsilon_1 + \varepsilon_2 + \eta$ which implies the assertion, since η was arbitrary small.

LEMMA 3. Under all assumptions of the lemma 1, the multifunction $F \cap G: X \to Y$ defined for all $x \in X$ by the formula:

(19) $(F \cap G)(x) := F(x) \cap G(x)$

is weakly lower semicontinuous.

PROOF (cf. [2], lemma 3): Let $\varepsilon > 0$ be fixed and let us select an $\sigma = \sigma(\varepsilon) > 0$ as in the lemma 1. Let V be any open neighbourhood of the point $x_0 \in X$. Since the function $f: X \to Y$ is continuous, hence there exists an open neighbourhood $\tilde{V} \subset V$ of our point x_0 such that:

(20) $G(\bar{x}) \subset K(G(\bar{\bar{x}}), \sigma)$ for $\bar{x}, \bar{\bar{x}} \in \tilde{V}$.

Bearing in mind that F is weakly lower semicontinuous we infer the existence of an open neighbourhood $U \subset \tilde{V}$ of the point x_0 and the existence of a point $x' \in U$ such that:

(21) $F(x') \subset K(F(x), \sigma)$ whenever $x \in U$.

By (21), (20) and (7) we obtain the inclusions:

(22) $F(x') \cap G(x') \subset K(F(x), \sigma) \cap K(G(x), \sigma) \subset K(F(x) \cap G(x), \varepsilon)$

whenever $x \in U$. That completes the proof.

LEMMA 4. Let X be a paracompact space, Y a Banach space and $F: X \to Y$ a weakly lower semicontinuous compact multifunction with compact, convex values.

Then F is $C_o(X, Y)$ - stable (cf. (i) and (ii)).

PROOF: The item (i) is an easy consequence of the theorem 1 and of the fact, that any continuous function with compact range is bounded. In order to prove the item (ii) observe, that by lemma 3 the multifunction

$$X \ni x \mapsto F(x) \cap \bar{K}(f(x), r + r_1) \subset Y$$

is weakly lower semicontinuous. Although in all infinite-dimensional Banach spaces the ball $\bar{K}(f(x), r + r_1)$ is never compact, but the intersection $F(x) \cap \bar{K}(f(x), r + r_1)$ is of necessary compact as well as oonvex. Invoking once again the theorem 1 we obtain a continuous mapping $g \in C_o(X, Y)$ being a desired selector for the above intersection. Thus the proof of $C_o(X, Y)$- stability of F is finished.

Now, we are in a position to state and prove our main result:

THEOREM 2. Let T and X be two paracompact spaces and $(Y, \|\cdot\|)$ a Banach space. Suppose that $F: T \times X \to Y$ is a multifunction such that:

(A) the set $F(t,x)$ is compact and convex for every $(t,x) \in T \times X$

(B) for every $x \in X$ the multifunction $F(\cdot, x)$ is weakly lower semicontinuous and for each of its continuous selectors g, one has:

(23) $$\lim_{u \to x} \sup_{t \in T} \operatorname{dist}(g(t), F(t,u)) = 0$$

Under such hypotheses there exists a continuous function $f: T\times X \to Y$ such that $f(t,x) \in F(t,x)$ whenever $(t,x) \in T\times X$.

PROOF: Let $C_o(T, Y) := C(T,Y) \cap B(T,Y)$ be a Banach space of bounded continuous functions equipped with the uniform norm (6).

Define the multifunction $H: X \to C_o(T, Y)$ by putting

(24) $H(x) := \{g \in C_o(T, Y): g(t) \in F(t, x) \text{ for each } t \in T\}$.

Taking into account that the range $F(T, x) := F(\cdot, x) * T$ is (for each $x \in X$) relatively compact in Y and hence bounded, we infer from theorem 1 that all values of H are nonvoid. If $g_1, g_2 \in H(x)$ then for $0 \le a \le 1$, $a \cdot g_1(t) + (1 - a)g_2(t) \in \in F(x,t)$ because of the convexity of all $F(t, x)$. Thus $ag_1 + (1-a)\, g_2$ belongs to $H(x)$ so that H is convex-valued. Moreover if $(g_n)_{n=1}^{\infty}$ is an uniformly convergent sequence of continuous functions from T onto Y such that $g_n(t) \in F(t,x)$ then $g := \lim_{n \to \infty} g_n \in H(x)$ as well and thus H is closed valued.

Let us prove that the multifunction H given by (24) is lower semicontinuous. To this end fix $g_o \in H(X)$ and $r > 0$. It is easily seen, that

(25) $$H^-(K(g_o, r)) := \{x \in X : H(x) \cap K(g_o, r) \ne \emptyset\} =$$

$$= \{x \in X : \text{there exists } g_1 \in C_o(T, Y) \text{ and } \varepsilon < r \text{ such that } g_1(t) \in F(t,x) \cap K_Y(g_o(t), r - \varepsilon) \text{ for all } t \in T\}.$$

Fix an $\varepsilon_o \in (0,r)$ and $x_o \in \bigcap_{t \in T} F(t, \cdot)^-(K(g_o(t), r - \varepsilon_o))$. Since $F(\cdot, x_o)$ is, by lemma 3, stable, it follows that there exists an $g_1 \in C_o(T, Y)$ such that :

(26) $g_1(t) \in F(t, x_o) \cap K(g_o(t), r - \varepsilon_o/2)$ for every $t \in T$.

From this fact and from (25) it fo ows that:

(27) $H^-(K(g_o, r)) = \bigcup_{0<\varepsilon<r} \bigcap_{t \in T} F(t, \cdot)^- (K(g_o(t), r-\varepsilon))$

To observe that $H^- (K(g_o,r))$ is open in X, let us fix a point

(28) $x_1 \in \bigcup_{0<\varepsilon<r} \bigcap_{t \in T} F(t, \cdot)^- (K(g_o(t), r - \varepsilon))$.

Therefore, the exists an $\varepsilon_1 \in (0, r)$ such that the intersection $F(t, x_1) \cap K(g_o(t), r - \varepsilon_1) \neq \emptyset$ is nonempty for every $t \in T$. Since the multifunction $T \ni t \longmapsto F(t, x_1) \subset Y$ is $C_o(T, Y)$-stable there exists an $g_2 \in C_o(T, Y)$ such that $g_2(t) \in F(t, x_1) \cap K(g_o(t), r - \varepsilon_2)$ for every $t \in T$, where $\varepsilon_2 \in (0, \varepsilon_1)$. By (23) there exists a neighbourhood $V(x_1)$ of x_1 such that dist $(g_2(t), F(t,x)) < \varepsilon_2/2$ for every $x \in V(x_1)$, $t \in T$.

Fix $x_2 \in V(x_1)$, $t_2 \in T$. Let $y_2 \in F(t_2, x_2)$ be such that $\| g_2(t_2) - y_2 \| < \varepsilon_2/2$. Since $\| g_2(t_2) - g_o(t_2)\| < r - \varepsilon_2$ we have $\| y_2 - g_o(t_2)\| \leq \| y_2 - g_2(t_2)\| + \| g_2(t_2) - g_o(t_2)\| <$ $< r - \varepsilon_2 + \varepsilon_2/2 = r - \varepsilon_2/2$ and so y_2 belong to $F(t_2,x_2) \cap K(g_o(t_2), r - \varepsilon_2/2)$. Thus we have proved that the neighbourhood $V(x_1)$ of x_1 is contained in the set $\bigcup_{\varepsilon \in (0, r)} \bigcap_{t \in T} F(t, \cdot)^-[K(g_o(t), r - \varepsilon)]$ which, therefore, is open. Let $h \in C(X, C_o(T, Y))$ be a continuous selector for H existing in compliance with celebrated Michael's selection theorem [9]. Define $f: T \times X \to Y$ by the formula $f(t,x) := h(x)(t)$. Since all functions of the family

$\{f(t,\cdot) : t \in T\}$ are equicontinuous, it follows that $f \in C(T\times X, Y)$ (cf. [1],[8]). Obviously, the function f is the claimed continuous selector for F. Now, we are going to replace in Ricceri's theorem 4.4 a selection theorem from [11] by the following comprehensive although somewhat complicated Michael's result ([10],[12]):

THEOREM 3 ([12]) Let X be a paracompact space, Y a Banach space, $Z \subset X$ a subset with $\dim_X Z \leq 0$, $C \subset X$ a countable subset, and $F: X \to Y$ a lower semicontinuous multifunction such that $F(x)$ is closed in Y for $x \notin C$ and $F(x)$ is convex for $x \notin Z$. Then F has a continuous selector.

Note, that $\dim_X Z \leq 0$ means that $\dim E \leq 0$ for every set $E \subset Z$ which is closed in X, where $\dim E$ denotes the covering dimension of E and observe that, for normal spaces X, $\dim_X Z \leq 0$ is valid if either $\dim Z \leq 0$ or $\dim X \leq 0$. Thus theorem 3 incorporates several known results, as surveyed in [10].

A direct modification of the proof of theorem 4.4 in [15], with theorem 3 invoked in the place of the result from [11] gives :

THEOREM 4. Let T and X be two paracompact topological spaces and Y a Banach space. Let $Z_1 \subset T$ and $Z_2 \subset X$ be two sets with $\dim_T Z_1 \leq 0$ and $\dim_X Z_2 \leq 0$ respectively and let $C_1 \subset T$ and $C_2 \subset X$ be two countable subsets. Suppose that a multifunction $F: T\times Y \to Y$ satisfies the following conditions:

(A-1) the set $F(t,x)$ is convex for every $(t,x) \in T\times(X - Z_2) \cup$

$\cup (T-Z_1)xX$ and closed for $(t,x) \in T \times (X-C_2) \cup (T-C_1) x X$,

(B-1) for every $x \in X$ the image $F(T, x) = F^x(T)$ is bounded, the multifunction $F^x := F(\cdot, x): T \to Y$ is lower semicontinuous and for each of its continuous selectors on has (23).

Under such hypotheses, for every closed set $D \subset X$ and every continuous selector $g_1: Tx\ D \to Y$ of the restriction $F | Tx\ D$ such that the functions of the family $\{g_1(t, \cdot) : t \in T\}$ are equicontinuous, there exists a continuous selector $f : Tx\ X \to Y$ for F such that:

(a) for every $x \in X$ the function $f^x := f(\cdot, x): T \to Y$ is continuous,

(b) the functions of the family $\{f_t := f(t, \cdot): t \in T\}$ are equicontinuous,

(c) the restriction $f | Tx\ D$ is equal to g_1.

REMARK 2: Recall a subset S of a topological space X is discrete if it has no accumulation point in X, and that C is sigma-discrete if C is a countable union of discrete sets S_n, $n \in N$. It is easy to check (cf. [12], p. 8)that theorem 3 (and thus also our theorem 4) remain valid with essentially the same proofs, if " countable " is weakened to "sigma--discrete" .

REMARK 3: Observe that theorem 3 cannot be directly applied to obtain the existence of a continuous selector of a multi-function F satisfying the hypotheses (A-1) and (B-1) of theorem 4. In fact, in theorem 4 condition (B-1) implies that F is jointly lower semicontinuous on the product space

Tx X but, as it is well-known, this product Tx X need not be paracompact.

REMARK 4: There exists a multifunction $F: R\times R \Rightarrow R$ (R denotes a real line) with compact, convex values, having lower semicontinuous all sections $F(t, \cdot)$, $t\in R$ and $F(\cdot, x)$, $x\in R$ but without any measurable selector.

PROOF: Let $h: R \to [0,1]$ be an arbitrary nonmeasurable function. Then put

$$(29)\qquad F(t, x) := \begin{cases} \{h(x)\} \text{ iff } & t = x \\ [0, 1] \text{ iff } & t \neq x. \end{cases}$$

It is easily checked that F defined by (29) fullfils all requirements. See also [16] for further interesting counterexamples.

Now we want to improve the theorem 4.5 from [15] in an analogous way. We say that a topological space X is extremally disconnected, if the closure of every open set is open. A multifunction $F:X \Rightarrow Y$ between topological spaces X and Y is upper semicontinuous if the set

$$(30)\quad F^{+}(U) := \{x\in X: \ F(x)\subset U\} = X - F^{-}(Y-U)$$

is open in X for any open set U in Y. Following [6] F is called closed, if the image $F(D) := \bigcup\{F(x): x\in D\}$ (cf.[5]) is closed in Y for every closed set D in X. A single-valued mapping f from X into Y is called compact if the fiber $f^{-1}(y)$ is compact in X for any $y\in Y$ and is called perfect, if it is continuous, closed and compact. Then Hasumi's [6] main theorem reads as follows:

THEOREM 5 ([6]) . Let X be an extremally disconnected

Hausdorff space, Y a regular Hausdorff space, and F an upper semicontinuous mapping from X into the family of all non-void compact subsets of Y. Then there exists a continuous selector $f: X \to Y$ for F. Furthermore we have the following:

(a-1) If the set $\{x \in X: y \in F(x)\}$ is compact in X for every $y \in Y$ then the selector f can be made compact

(a-2) If F is also closed, then f can be made closed and compact, so that f is perfect.

Combining theorem 5 with theorem 3 we obtain the following analogue of Ricceri's theorem 4.5 from [15].

THEOREM 6. Let T be an extremally disconnected Hausdorff topological space and let X, C, Z and Y be as in the theorem 3. Suppose that the multifunction $F: T \times X \to Y$ has the following properties:

(A-2) the set $F(t,x)$ is compact for every $(t,x) \in T \times (X-C)$ and convex for every $(t,x) \in T \times (X-Z)$

(B-2) for every $c \in C$ the multifunction $F^c: T \to Y$ defined by $F^c(t) := F(t,c)$ has a continuous selector,

(C-2) for every $x \in X$ the set $F(T, x)$ is bounded, the multifunction $F(\cdot, x): T \to Y$ is upper semicontinuous and for each of its continuous selectors g on has (23).

Under such hypotheses, the thesis of theorem 4 holds.

REMARK 5: In theorem 6, in general, the multifunction F is neither lower semicontinuous nor upper semicontinuous on the product space TxX. For more informations about such multifunctions with upper semicontinuous X-sections and lower semicontinuous T-sections the reader is refered to [16], where a multivalued analogue of famous Kempisty's theorem is presented. Note, that many results about so-called Caratheodory's selectors (see [7], [14])may be improved by using the recent Michael's theorem 3.

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O CIĄGŁYCH SELEKTORACH DLA MULTIFUNKCJI OKREŚLONYCH NA PRZESTRZENIACH PRODUKTOWYCH

Streszczenie

Udowodniono istnienie ciągłego selektora dla multifunkcji dwóch zmiennych, której jedne cięcia są słabo półciągłe z dołu w sensie Myjaka i de Blasi, a drugie półciągłe z dołu, i która ponadto spełnia pewien dodatkowy warunek. W dalszym ciągu wskazano na możliwość wzmocnienia pewnych kryteriów Ricceriego [15] w efekcie użycia ogólniejszego twierdzenia Michaela z [12] w miejsce jego wcześniejszego wyniku [11] zastosowanego w dowodach z [15].