Motion Analysis of Timing Belt Used in Control Systems

Domek Grzegorz[*]

Department of Mathematics, Physics and Technology, Kazimierz Wieki University, Bydgoszcz, Poland

Abstract The paper presents the results of measurements and analyzes gear timing belts, used in control systems. Many of the design features of timing belt determines its usefulness in such systems. Knowledge of technical parameters allows for the proper use of such belts in the construction.

Keywords: timing belts in control system, gear with timing belts

1. Introduction

Mechanical control system depends largely on the mechanisms used and drives. Electronic systems have the time and speed of response, and data transmission [1]. By using the latest design features are well known and can use this knowledge in the design process. The situation is different in the case of mechanisms, it seems that their characteristics have been established a long time ago. Despite intensive development of mechanical transmissions, there is no current descriptions and research. This applies particularly to the transmission of timing belts, which are more and more applications in the control systems and control [2,3].

2. Timing Belt in Control Systems

One of the oldest applications is the timing in internal combustion engines [9]. Due to the limited knowledge of toothed belts, some manufacturers have returned to use in this application chains, while others intensely developed these gears using such non-circular wheels (Figure 1).
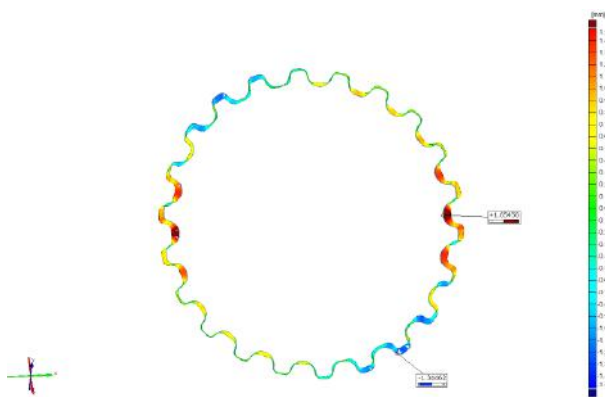

Figure 1. Non-circular pulley from valve timing of the combustion engine

Increase the speed of opening and closing valves, gave the savings in fuel consumption, exhaust emissions and reduced engine noise emissions. The use of timing belts in electric servo steering system has also contributed to a reduction in fuel consumption of vehicles. Currently, the number of gear timing belts, in motor vehicles approaching twenty [10].
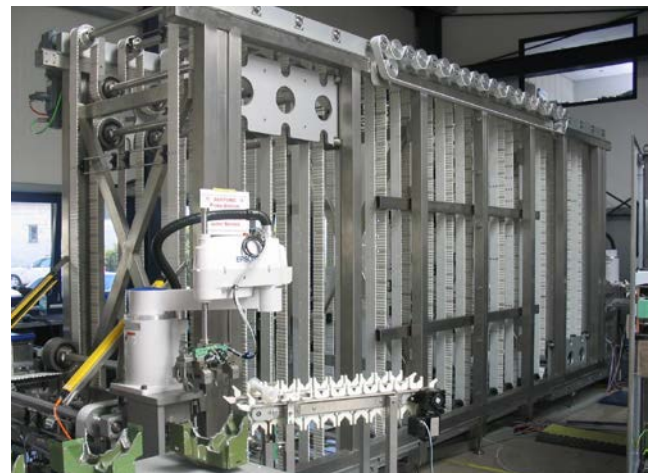

Figure 2. Storage vials in pharmaceutical factory

A more rapid development can be observed in industrial applications. The accuracy of movement in the drive axis CNC machines robots and manipulators resulted in the need to increase the quality and accuracy of production of toothed belts. The entire group of such systems can only drive belts made to the highest standard of precision.

Presented "magazine vials in motion" show that the inaccuracy of the carrier must be compensated straps (Figure 2). Another example is the bottling system whose construction involves filling 40 thousand. bottles per hour. In such applications the standard belt does not achieve the desired accuracy of the motion and speed of 20 thousand. bottles per hour (Figure 3).

Very important is the choice of strips made of the right materials. In the presented application creep caused by significant inaccuracies of motion and rapid damage the belt (Figure 4). Therefore, in this case, the selected edge solution. (color belt means a belt newest generation) [11].
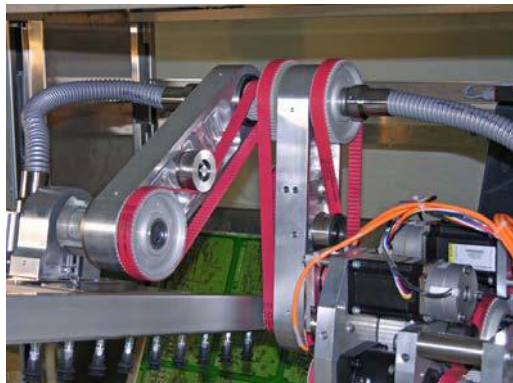
Figure 3. Control system of bottle filling system
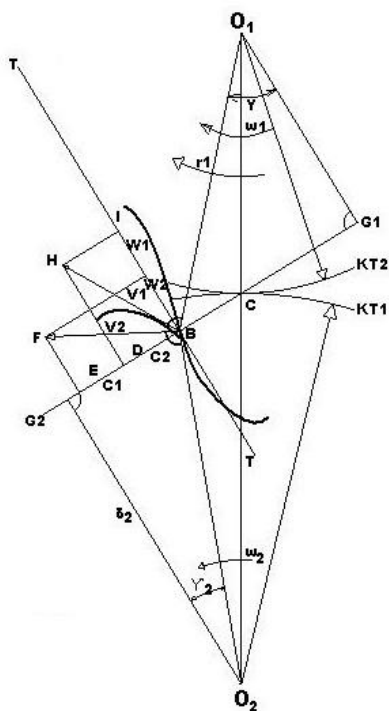

Figure 4. Timing belts i robotics

Figure 5. Willis principle

Analysis of the kinematics of the toothed belt started from the comparison with the chains. This sufficed in the first constructions belts of trapezoidal teeth. They differed only on the properties of the cord chain which was not elastic. In subsequent constructions belts rounded teeth and began to look for analogies to helical gears. Perpendicular at the point where mating teeth "B" runs

through the central point of engagement "C" (Figure 5). Trajectories targeted tooth movement in relation to the wheel usually rays, because the curve of the helical known not suitable for the description. The main advantage of such an engagement is lack sliding friction, gear teeth sweep up after yourself. This is the kind of cooperation it seeks in gear with the timing belts, as limited by this wear volume of teeth due to friction in the process of conjugation [12,13].

The problem is also belt drives internal friction. Can be reduced by reducing the waist under the neutral axis, while increasing the susceptibility of bent. Reduce the amount of material for the carrier layer, it can also be accomplished by the use of toothed belts have the same pitches while a different type of cords [14].

3. Description of the Belt Tooth Movement

The gear units with timing belts, can be distinguished equivalent focal point meshing, but the point "C" is moving with the movement of the wheels. Calculating the Lagrange difference kinetic and potential energy (Figure 6).

$$
\begin{aligned}
L = K - P &= \frac{1}{2}\{mq_1^2 + \frac{J_B}{r_B^2}(r_M\Omega + q_2')^2 \\
&+ (J_M + m\sigma^2)\Omega^2 + 2m\sigma q_1'\Omega\cos(\Omega t)\} \\
&- \frac{1}{2}\{C_M q_1^2 + C_C(-q_2 - q_1\cos a)^2 \\
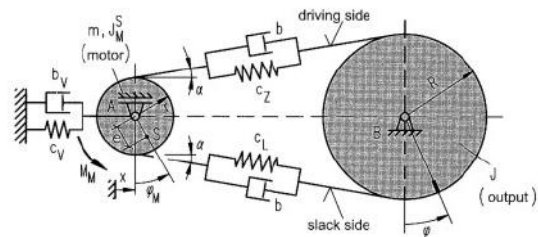&+ C_B(q_2 - q_1\cos a)^2\}
\end{aligned} \tag{1}
$$

Figure 6. Schematic representation of the power transmission with timing belt [8]

$$
\frac{\partial L}{\partial \dot{q}_1} = \frac{1}{2}\{2m\dot{q}_1 + 2m\sigma\Omega\cos(\Omega t)\} = m\dot{q}_1 + m\sigma\Omega\cos(\Omega t)
$$

$$
\frac{\partial L}{\partial \dot{q}_2} = \frac{1}{2}\{\frac{J_B}{r_B^2}2(r_M\Omega + \dot{q}_2)\} = \frac{J_B}{r_B^2}(r_M\Omega + \dot{q}_2)
$$

$$
\frac{d}{dt}\begin{bmatrix} \dfrac{\partial L}{\partial \dot{q}_1} \\ \dfrac{\partial L}{\partial \dot{q}_2} \end{bmatrix} = \begin{bmatrix} mq_1'' + m\sigma\Omega^2\sin(\Omega t) \\ \dfrac{J_B}{r_B^2}q_2'' \end{bmatrix}
$$

$$
= \begin{bmatrix} m & 0 \\ 0 & \dfrac{J_B}{r_B^2} \end{bmatrix}\begin{bmatrix} q_1'' \\ q_2'' \end{bmatrix} - \begin{bmatrix} m\sigma\Omega^2\sin(\Omega t) \\ 0 \end{bmatrix}
$$

$$\frac{\partial L}{\partial \dot{q}_1} = -\frac{1}{2}\left\{2C_M q_1 + 2C_C(q_2 + q_1 \cos a)\cos a + 2C_B(q_2 - q_1 \cos a)(-\cos a)\right\}$$

$$= -C_M q_1 - C_C q_2 - C_C q_1 \cos^2 a + C_B q_2 \cos a - C_B q_1 \cos^2 a$$

$$= \left\{-C_M - (C_C + C_B)\cos^2 a\right\}q_1 - \left\{(C_C - C_B)\cos a\right\}q_2$$

$$\frac{\partial L}{\partial \dot{q}_2} = \frac{1}{2}\left\{2C_C(q_2 + q_1 \cos a) + 2C_B(q_2 - q_1 \cos a)\right\}$$

$$= -C_C q_2 - C_C q_1 \cos a - C_B q_2 + C_B q_1 \cos a$$

$$= -(C_C - C_B)q_1 \cos a - (C_C + C_B)q_2$$

after the above analysis we get:

$$\begin{bmatrix} \dfrac{\partial L}{\partial \dot{q}_1} \\ \dfrac{\partial L}{\partial \dot{q}_2} \end{bmatrix} = \begin{bmatrix} -\{C_M - (C_C + C_B)\cos^2 a\} - (C_C - C_B)\cos a \\ -(C_C - C_B)q_1 \cos a - (C_C + C_B) \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Lagrange equation takes the form:

$$\begin{bmatrix} m & 0 \\ 0 & \dfrac{J_B}{r_B^2} \end{bmatrix}\begin{bmatrix} q_1'' \\ q_2'' \end{bmatrix} - \begin{bmatrix} m\sigma\Omega^2\sin(\Omega t) \\ 0 \end{bmatrix}$$

$$= -\begin{bmatrix} \{C_M - (C_C + C_B)\cos^2 a\}(C_C - C_B)\cos a \\ (C_C - C_B)q_1 \cos a(C_C + C_B) \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad (2)$$

Transmission dynamics occurring variability depends on the size and mass of the individual teeth. Possible collision belt teeth and wheels introduce additional vibration, and that it should be used to limit the right materials for the backing layer to strip underwent the smallest elongation. Should analyze the geometric form of rebate in a circle and the tooth at the waist [4,5]. The beginning and the end of cooperation should take place without mutual slip. Assuming that managed to reduce the tendon elongation, in the case of one scale can be omitted to describe the process of engaging the belt wheel, you can use the Lagrange description of the pendulum with variable coupling point (Figure 7).
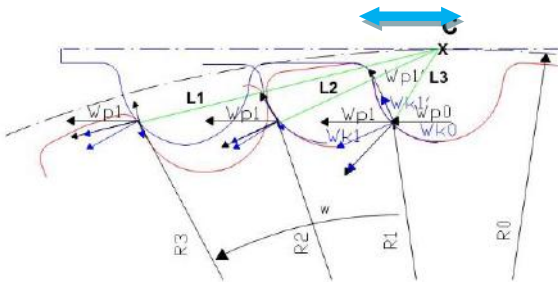

Figure 7. Movement of the teeth in the coupling

Comparing the process of engagement of the belt tooth with the wheel pendulum movement with variable length vibration equation can be written to this step of the form:

$$\frac{d}{dt}[m(l_0 + \Delta l)^2 \dot{\varphi}] + mg(l_0 + \Delta l)\varphi = 0 \qquad (3)$$

or:

$$m(l_0 + \Delta l)^2 \ddot{\varphi} + 2m(l_0 + \Delta l)\Delta l\dot{\varphi} + mg(l_0 + \Delta l)\varphi = 0 \quad (4)$$

In the present equation, m is the mass of a tooth-belt, l_0 - distance from the center of the tooth belt from the central point of engagement "C", Δl- value scale belt or wheel (depending on whether the movement takes the wheel or belt).

It is very important for the proper conduct of the process is to maintain the proper pitch diameter which in these transmissions is measured at the height of the neutral axis of the carrier layer [6]. Changing the position of the belt or cord diameter changes to unfavorable feedback. This also applies to the diameter of the surfaces on which they are co-belt wheels and in part, the diameter of the gear tooth tips of the feet in diameter part [7].

In order to improve the liquidity of engagement should first analyze the coverage ratio of teeth in the process of conjugation. While maintaining a small belt pitch while increasing the number of teeth of the pulley is obtained a situation in which several tooth is involved in the coupling to the wheel.

$$X = \frac{\sqrt{2D_p h_i - 4h^2 - h_i^2}}{2P} \qquad (5)$$

X-coverage factor D_p-foot diameter pulley teeth, tooth height, h_i-axis of the cord, h-height of the belt tooth.
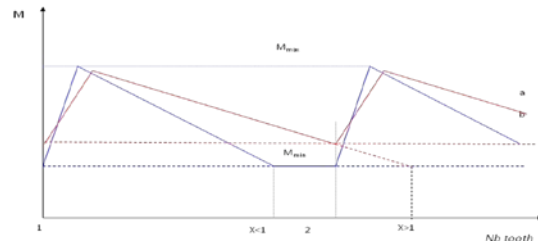

Figure 8. Changing the torque caused by the coupling

Was carried out a series of studies on uniformity of transmission operation. Variability called coverage ratio is a primary cause of this phenomenon. Requested again to solutions similar to cylindrical gears and timing belts used helical, herringbone and the arc. There are not conducted a more detailed analysis of problems associated with the construction of a timing belt and these structures solved the problem of smoothness and noise. Unfortunately, these bands are much worse perform tasks power transmission and control. The total deformation of teeth on the arc of contact depends also on geometric properties of the belt, such as the pitch utilization factor. The more comprehensive coupling model can be expressed in form of the following formula:

$$\frac{S_1}{S_2} = f(\sigma_k, \sigma_p, K_W, A_{kp}, Y, Z) \qquad (6)$$

where: K_W belt pitch utilization factor, σ_k cord deformation (extension and twist), σ_p belt material deformation causing belt tooth height change σ_{ph} and the width change σ_{pb} as well as shape change σ_{pA}, A_{kp} adhesion factor for cord, belt material and additional materials, Y the toothed belt pitch to toothed pulley pitch ratio, Z – belt and pulley wear of volumetric Z_v and energetic Z_e type.

$$\sigma_p = \{\sigma_{ph}; \sigma_{pb}; \sigma_{pA}\} \qquad (7)$$

$$Z = Z_v + Z_e \qquad (8)$$

Use of toothed belts of the same pitch value and different cord types allows to satisfy the need for internal friction reduction (by reducing the tooth height and the height below the neutral axis) with simultaneous increase of flexibility and making use of flat belt advantages.

When the coupling is expressed in form of the relationship of temporary stress values, it takes the form:

$$\frac{dS_1}{dS_2} = f(\mu_{pk}, X_c, X_B, K_w, K_z, F_N, \varepsilon^N, d_u, dz_{co}, dz_{bo}) \quad (9)$$

The temporary number of teeth on the arcs of contact of the driving pulley dz_{co} and the driving pulley dz_{bo} respectively, significantly influences the change of coupling, particularly for transmission gears, where the value of X, determined from the formula (5) does not exceed the unity. Belt material properties specified by the formula (6), indicating to the influence of deformations on the change of material properties are also important. The pre-tensioning force F_N, influences the coupling character by influence on matching the belt and the pulley pitches as well as on the angle – α_0, the value of which depends on the mechanical properties of the load-carrying layer [4].

4. Conclusion

Gears of timing belts, excellent excel in the control and regulation systems still standing before them a wide field of applications. Understanding the structural design allows the distinction of belts and wheels: driving from controls. Like other mechanical transmissions used in the control also in this case is selected with a gear limited backlash or "backlash free". Given the problems described should be aware of reduced efficiency and inaccuracies. Value of initial stress force is essential in process of inequality of work of the shaft. On the basis introduction of small changes of the force it can be stated that its conditioning to geometric parameters of the belt as well as to the value of circumferential force will allow to develop operating parameters of shafts with timing belt. Phenomena associated with the contact between the toothed belt and the pulley can be divided into categories. The first category includes phenomena occurring inside the belt and is associated with load transfer from the belt material to the cord as well as effects occurring between respective belt and pulley surfaces. In some experiments synchronous gear worked parallel with standard belt gear in order to improve power transmission trough friction. Analysis of those effects constitutes the grounds for individual attitudes to design and operation of toothed belt transmission gears.

References

[1] Domek G., Leistungverluste in Zahnriemengetrieben, Antriebstechnik 12/2006, s. 30-31.

[2] Domek G., Malujda I., Modeling of timing belt construction, Wiley Inter Science PAMM Volume 7, Issue 1, Date: December 2007, Pages: 4070045-4070046.

[3] Domek G., Dudziak M., Energy dissipation in timing belts made from composite materials, Journal of Advanced Materials Research, Vols. 189-193(2011) pp 4414-4418.

[4] Domek G., Meshing model in gear with timing belt, Journal of Advanced Materials Research, Vols. 189-193(2011) pp 4356-4360.

[5] Domek G., Meshing in gear with timing belts, International Journal of Engineering and Technology (IJET), vol. 3, no. 1, pp. 26-29, 2011.

[6] Domek G., Meshing model in gear with timing belt, Journal of Advanced Materials Research, Vols. 189-193(2011) pp 4356-4360.

[7] Domek G., Timing belts dynamics model approach, Journal of Mechanics Engineering and Automation, 2012, Vol.2 N.8, p 495-497.

[8] Dressing H., Holzweissig F., Dynamics of Machinery, Theory and Applications, Springer Verlag, Berlin Heidelberg 2010.

[9] M. Dudziak, New aspects od driving rubber belts life determination, *Zagadnienia Eksploatacji Maszyn*, 1993, zeszyt 4, 455-469.

[10] M. Dudziak, About internal friction problems and energy dissipation in rubber power transsmision belts, 1990,*Rozprawy*, nr 229, p. 167, Politechnika Poznańska.

[11] M. Dudziak, Directions in development of flexible connector belts design, In: *Modelling and Simulation in Machinery Productions*, Proceedings of Inter. Conference „Modelling and Simulation in Machinery Productions" 1997, Puchov, Slovakia.

[12] M. Dudziak, G. Domek, , Mechanics of bending of timing belts with non straight teeth, *The Tenth Pan American Congress of Applied* Mechanics, *X PACAM' 08*, 215-218, Cancun 2008.

[13] M. Dudziak, G. Domek, Model of load in timing belts, *The Tenth Pan American Congress of Applied Mechanics, X PACAM' 08*, 219-222, Cancun2008.

[14] M. Dudziak, G. Domek, A. Kołodziej, Modelling of constructional feautures of timing belts made of materiale with macromolecular structures, *XI PACAM"10*, Sao Carlos, SP, Brasil2010.