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Description of Timing Belt Coupling Using a Pendulum with Variable Coupling Point

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Abstract

In the description of the process of coupling of pulleys with the toothed belt often existing solutions are used. Depending on the application they are similar to flat belt, chain or hellical gears. Euler made a first approximation of the coupling by ratio of the stress of active and passive tendon in the form of an exponential function of the natural logarithm base. The exponent of this function contained a number of factors related to the material of belt and pulleys. For the toothed belt transmissions, in which there is slack in the small loads such description is sufficient. Tension in timing belts are lightweight and very stretchy while the coupling between the belt and the pulley is a friction. After crossing the frictional load coupling occuring between the pulley teeth and belt becomes geometrical. The coupling takes place at successive coupling of the teeth and its nature depends on both the material and the geometry of the teeth. The most appropriate process used for a description of the motion is a pendulum with variable coupling point.

Keywords: timing belts, coupling in belt drive.

1. Introduction

The geometrical features of timing belts and toothed pulleys have to assure the best conditions of coupling process. The shape of the notch of the pulley is not the simple reflection of the shape of tooth of the belt. After coupling with the pulley, the belt is further squeezed, curved and it slides on the pulley. These processes are the basic reason of deformation and the wear of the teeth of the belt. One should take into account dynamic strengths connected with accelerations and

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delays in the transmission and their inner strengths connected with the linear speed.

Mechanical control system depends largely on the mechanisms that are applied and drives. Electronic systems have the time and speed of response, and data transmission [Domek G., 2006]. By using the latest design features belts can solve many constructional problems. The situation is different in the case of classical mechanical gears where it seems that their characteristics have been established a long time ago. Despite intensive development of mechanical transmissions, there is no current descriptions and research. This applies particularly to the transmission of timing belts, which are used in more and more applications as well as in the control systems and control [Domek G., Malujda I., 2007, Domek G., Dudziak M., 2011].

2. Timing belt applications

One of the oldest applications is the timing in internal combustion engines [Dudziak M., 1993]. Due to the limited knowledge of toothed belts, some of manufacturers returned to using chains in this application, while others intensely developed the gears using for example non-circular pulleys (Fig.1).

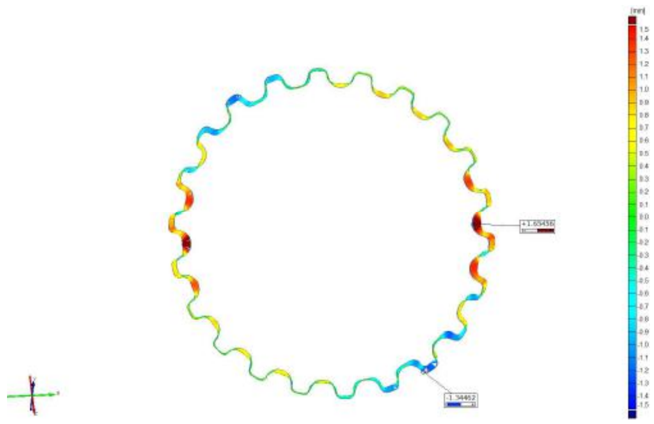

Fig. 1. Non-circular pulley from valve timing of the combustion engine.

Increasing of speed in opening and closing of valves resulted in savings in fuel consumption, reduction of exhaust emissions and limited engine noise emissions. The use of timing belts in electric servo steering system has also contributed to reduction in fuel consumption of vehicles. Currently, the number of gear timing belts, in motor vehicles approaches twenty [Dudziak M., 1990]. More rapid development can be observed in industrial applications. The accuracy of movement in the drive axis of CNC machines robots and manipulators resulted in the necessity to increase the quality and accuracy of production of toothed belts

(Fig.2). The entire group of such systems can be run only by drive belts made accoring to the highest standard of precision. Presented storage of vials in motion shows that the inaccuracy of the carrier must be compensated by additional belts. Another example is the bottle filling system whose construction allows for filling of 40 thousand bottles per hour. In such applications a standard belt does not achieve the desired accuracy of the motion and its maximum speed does not exceed 20 thousand bottles per hour.

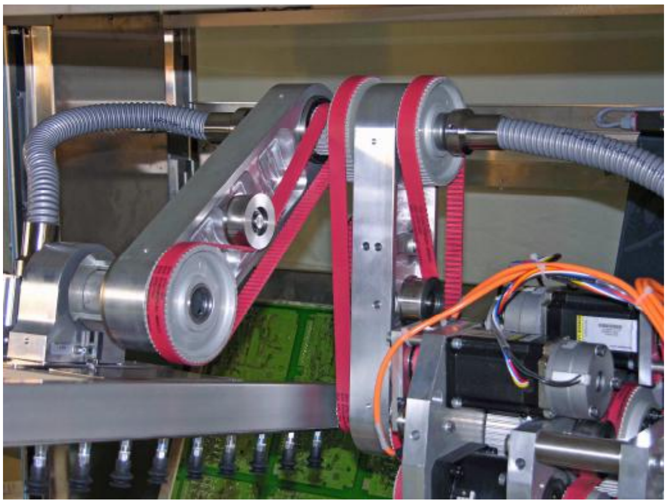

Fig. 2. Timing belts in robotics [15].

The choice of right material of the belt is a key factor. In the presented applications creep is the main cause of inaccuracy of motion and rapid damage the belt (Fig.2). Therefore, in this case, the selected belt was specially designed for extreme load application [Dudziak M., 1997].

3. Description of the belt tooth movement

The main method for designing toothed belts is to select appropriate belt and cord material. The cord extension value on the arc of contact is determined by the angle of the arc of contact over which the belt teeth are deformed. The number of deformed teeth depends also on the value of the pitch [. Dressing H., Holzweissig F., 2010]. To develop a qualitative model of the geometric coupling between the toothed belt and pulleys, it is necessary to consider deformation and the number of coupled belt teeth. This type of cooperation between belt and pulley is similar to chain and typical for old types of gears (Fig. 3). Belt is supported by top of head of pulley teeth. So far, deformation of material between cord and pulley is not taken to model and different high of cord on arc of contact can not be described. It has been empirically confirmed that periodical deformation of the belt cross-section during bending on the pulley significantly influences the value of energy loss due to internal friction in the belt material and rises the belt temperature [Domek G., Malujda I., 2007, Domek G., 2012]. The highest value of internal friction and energy dissipation occurs in the compressed layers of the belt below the neutral axis. Main friction types between the toothed belt and the pulley are associated with belt movement within the tooth space as well as the coupling and decoupling process. The above mentioned conditions contributed to the new interpretation of phenomena taking place during combined form-fitting and friction coupling as well as to the directions of the toothed belt structure development.

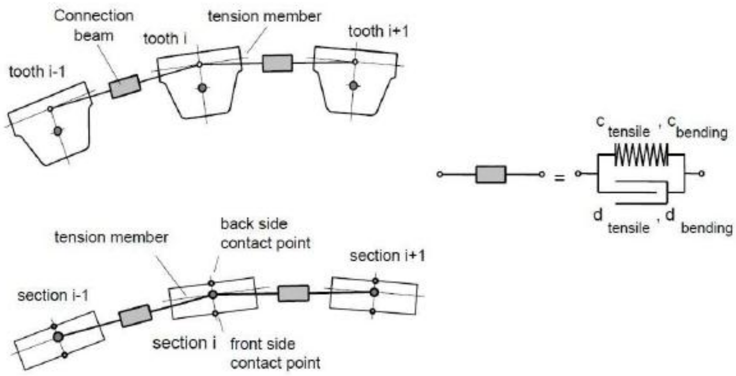

Fig. 3. Rigid body representation of synchronous and non-toothed belt [6]

Analysis of the kinematics of the toothed belt started from comparison to the chains. This was sufficient for description of the first belts with trapezoidal teeth. They were different only in belt and chain materials. In subsequent constructions belts started to have rounded teeth and began to look analogical to helical gears. Perpendicular at the point where touching teeth “B” run through the central point of engagement “C” (Fig.5). Target trajectory of tooth movement in relation to the pulley usually fails because the curve known from helical gears are not suitable for the description. The main advantage of such an engagement is lack sliding friction, gear teeth sweep up on each other. Description of such a cooperation is still seeked. In timing belt gears understanding of this phenomena may limit volume wear of teeth due to friction in the process of conjugation [Dudziak M., Domek G., 2008a, Dudziak M., Domek G., 2008b].

In gear units with timing belts, one can distinguish equivalent local point meshing “B”. However, point “C” is moving with the movement of the pulleys.

Calculating the Lagrange difference kinetic and potential energy (Fig.4).

$$L = K - P = \frac{1}{2}\{mq_1^2 + \frac{J_B}{r_B^2}(r_M\Omega + \dot{q}_2)^2 + (J_M + m\sigma^2)\Omega^2 + 2m\sigma\dot{q}_1\ \Omega\cos(\Omega t)\}$$
$$-\frac{1}{2}\{C_M q_1^2 + C_C(-q_2 - q_1 cos\alpha)^2 + C_B(q_2 - q_1 cos\alpha)^2\} \quad (1)$$

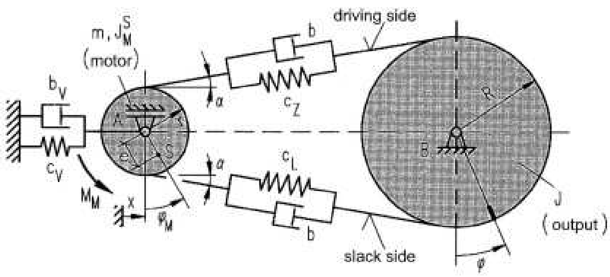

Fig. 4. Schematic representation of the power transmission with timing belt [Dressing H., Holzweissig F., 2010]

$$\frac{\partial L}{\partial \dot{q}_1} = \frac{1}{2}\{2m\dot{q}_1 + 2m\sigma\Omega\cos(\Omega t)\} = m\dot{q}_1 + m\sigma\Omega\cos(\Omega t) \quad (2)$$

$$\frac{\partial L}{\partial \dot{q}_2} = \frac{1}{2}\{\frac{J_B}{r_B^2}2(r_M\Omega + \dot{q}_2)\} = \frac{J_B}{r_B^2}(r_M\Omega + \dot{q}_2) \quad (3)$$

$$\frac{d}{dt}\begin{bmatrix}\frac{\partial L}{\partial \dot{q}_1}\\ \frac{\partial L}{\partial \dot{q}_2}\end{bmatrix} = \begin{bmatrix} m\ddot{q}_1 + m\sigma\Omega^2\sin(\Omega t)\\ \frac{J_B}{r_B^2}\ddot{q}_2\end{bmatrix} = \begin{bmatrix} m & 0\\ 0 & \frac{J_B}{r_B^2}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} - \begin{bmatrix} m\sigma\Omega^2\sin(\Omega t)\\ 0\end{bmatrix} \quad (4)$$

$$\begin{aligned}\frac{\partial L}{\partial \dot{q}_1} &= -\frac{1}{2}\{2C_M q_1 + 2C_C(q_2 + q_1 cos\alpha)\ cos\alpha + 2C_B(q_2 - q_1 cos\alpha)(-cos\alpha)\}\\ &= -C_M q_1 - C_C q_2 - C_C q_1 cos^2\alpha + C_B q_2 cos\alpha - C_B q_1 cos^2\alpha\\ &= \{-C_M - (C_C + C_B)cos^2\alpha\}q_1 - \{(C_C - C_B)cos\alpha\}q_2\end{aligned} \quad (5)$$

$$\begin{aligned}\frac{\partial L}{\partial \dot{q}_2} &= \frac{1}{2}\{2C_C(q_2 + q_1 cos\alpha) + 2C_B(q_2 - q_1 cos\alpha)\} = -C_C q_2 - C_C q_1 cos\alpha - C_B q_2 + C_B q_1 cos\alpha\\ &= -(C_C - C_B)q_1 cos\alpha - (C_C + C_B)q_2\end{aligned} \quad (6)$$

$$\begin{bmatrix}\frac{\partial L}{\partial \dot{q}_1}\\ \frac{\partial L}{\partial \dot{q}_2}\end{bmatrix}=\begin{bmatrix}-\{C_M-(C_C+C_B)cos^2\alpha\} & -(C_C-C_B)cos\alpha\\ -(C_C-C_B)q_1cos\alpha & -(C_C+C_B)\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix} \tag{7}$$

Lagrange equation takes the form:

$$\begin{bmatrix}m & 0\\ 0 & \frac{J_B}{r_B^2}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}-\begin{bmatrix}m\sigma\Omega^2\sin(\Omega t)\\ 0\end{bmatrix}=-\begin{bmatrix}\{C_M-(C_C+C_B)cos^2\alpha\} & (C_C-C_B)cos\alpha\\ (C_C-C_B)q_1cos\alpha & (C_C+C_B)\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix} \tag{8}$$

Variability of transmission dynamics occurring depends on the size and mass of the individual teeth. Possible collision of belt teeth and pulleys introduces additional vibration. In order to limit it, right material should be used for the cord in order to reduce elongation. One should also analyse the geometric form of tooth in timing pulley [Domek G., 2011a, Domek G., 2011b]. The beginning and the end of cooperation should take place without mutual slip. Assuming that one managed to reduce the cord elongation to minimum or the scale of the elongation is omitted, than to describe the process of engaging the belt and pulley, one can use the Lagrange description of the pendulum with variable coupling point (Fig. 5).

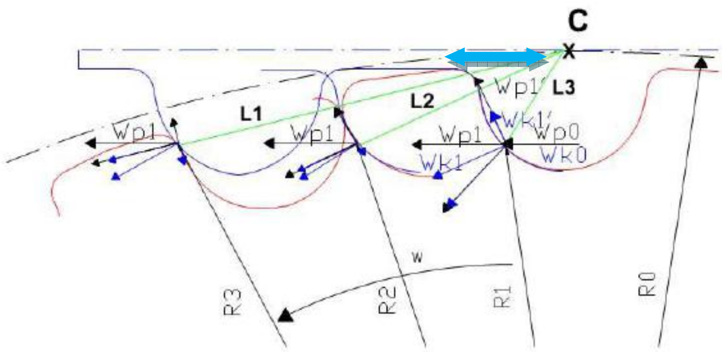

Fig.5. Movement of the teeth in the coupling.

Comparing the process of engagement of the belt tooth with the pulley pendulum movement with variable length vibration equation can be written to this step in the form:

$$\frac{d}{dt}\left[m(l_0+\Delta l)^2\dot{\varphi}\right]+mg(l_0+\Delta l)\varphi=0 \tag{9}$$

or:

$$m(l_0+\Delta l)^2\ddot{\varphi}+2m(l_0+\Delta l)\Delta l\dot{\varphi}+mg(l_0+\Delta l)\varphi=0 \tag{10}$$

In the present equation, *m* is the mass of a tooth-belt, l_0 - distance from the center of the tooth belt from the central point of engagement *"C"*, *Δl*- value scale belt or pulley (depending on whether the movement takes the pulley or belt).

It is very important for the proper conduct of the process to maintain the proper pitch diameter which in these transmissions is measured at the height of the neutral axis of the carrier layer [Domek G., 2011c]. Changing the position of the belt or cord diameter changes to unfavorable cooperation. This also applies to the diameter of the surfaces on which belt and pulley are contacting [Domek G., 2012]. In order to improve the liquidity of engagement one should first analyze the coverage ratio of teeth in the process of conjugation. Using small pitches increases the number of teeth of the pulley that are involved in the coupling with the belt.

$$X = \frac{\sqrt{2D_p h_l - 4h^2 - h_l^2}}{2P} \tag{11}$$

X-coverage factor, D_p-foot diameter pulley teeth, tooth height, h_l-axis of the cord, *h*-height of the belt tooth.

A series of studies were carried out on uniformity of transmission operation. Variability called coverage ratio is a primary influences this phenomenon. Reffering to solutions similar to hellical gears, in timing belt herringbone and the arc form of teeth are used. Usage of these teeth forms was not preceeded by any detailed analysis and therefore it only solved the problem of smoothness and noise. Unfortunately, such belts are much worse in performing tasks of power transmission and control. The total deformation of teeth on the arc of contact depends also on geometric properties of the belt, such as the pitch utilization factor. More comprehensive coupling model can be expressed in form of the following formula:

$$\frac{S_1}{S_2} = f(\sigma_\mathrm{k}, \sigma_\mathrm{p}, K_\mathrm{W}, A_\mathrm{kp}, Y, Z) \tag{12}$$

where: KW belt pitch utilization factor, σk cord deformation (extension and twist), σp belt material deformation causing belt tooth height change σph and the width change σpb as well as shape change σpA, Akp adhesion factor for cord, belt material and additional materials, Y the toothed belt pitch to toothed pulley pitch ratio, Z – belt and pulley wear of volumetric Zv and energetic Ze type.

Use of toothed belts of the same pitch value and different cord types allows to satisfy the need for internal friction reduction (by reducing the tooth height and the height below the neutral axis) with simultaneous increase of flexibility and making use of flat belt advantages.

4. Conclusions

The most approximate description of the timing belt movement is a motion of pendulum with variable coupling point. Described problems should be aware of reduced efficiency and inaccuracies of power transsmision. Phenomena associated with the contact between the toothed belt and the pulley can be divided into categories. One of them includes phenomena occurring inside the belt and is associated with load transfer from the belt material to the cord as well as effects occurring between respective belt and pulley surfaces. In some experiments synchronous gear worked parallel with standard belt gear in order to improve power transmission trough friction. Analysis of those effects constitutes the grounds for individual attitudes to design and operation of toothed belt transmission gears. Gears with timing belts are excellent in control and regulation systems but they are still standing before wide field of applications.

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