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PROGRAMMING OF COMPOSITE PLATES DAMAGE CALCULATION

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Abstarct: *The goal of this paper is to present the numerical results of elastic damage of thin unidirectional fiber-reinforced composite plates. The numerical implementation uses a layered shell finite element based on the Kirchhoff plate theory. Newton-Raphson method is used to solve the system of nonlinear equations and evolution of damage has been solved using return-mapping algorithm. The analysis is performed by finite element method and user own software is created in MATLAB programming language. One problem for two different materials was simulated in order to study the damage of laminated fiber reinforced composite plates.*

Keywords: *continuum damage mechanics, composite plate, finite element method*

3. INTRODUCTION

Composite materials are now common engineering materials used in a wide range of applications. They play an important role in the aviation, aerospace and automotive industry, and are also used in the construction of ships, submarines, nuclear and chemical facilities, etc.

The needs of explicit safety requirements for development of the Processes of design, manufacturing, maintenance, operation and requirement s are tightly linked to change and innovation. The challenges derived from new materials, new processes and new structural concepts are inherent to the use of composites for light structures. It is true that it is the responsibility of each individual discipline to implement processes that ensure the meeting of safety objectives. The damage to do so, for the most part results in structural integrity loss, so a coordinated set of requirements must be put in place, and monitored and enforced.

The meaning of the word damage is quite broad in everyday life. In continuum mechanics the term damage is referred to as the reduction of internal integrity of the material due to the generation, spreading and merging of small cracks, cavities and similar defects. In the initial stages of the deformation process the defects (microcracks, microcavities) are very small and relatively uniformly distributed in the microstructure of the material. If the damage reaches the critical level (depends on the load type and material used), subsequent growth of defects will concentrate in some of the defects already present in material [7]. Damage is called elastic, if the material deforms only elastically (in macroscopic level) before the occurrence of damage, as well as during its evolution. This damage model can be used if the ability of the material to deform plastically is low. Fiber-reinforced polymer matrix composites can be considered as such materials. Composites represents a family of structural materials for which the accumulation of structural damage is complicated process. It involves fatigue damage initiation, damage growth contributions, continued accidental damage occurrences and, in addition, the contributions from property changes due to materials and manufacturing process failures.

Unidirectional composites reinforced by long fibers are one kind of multicomponent composites. They can be considered as orthotropic and it is not necessary to perform homogenization in structural analysis. If multidirectional fiber-reinforced composites are analyzed, it is necessary to be perform homogenization in order to calculating the components of constitutive matrix, or tensor [15,8], which are required for further analysis of composite structures, for example damage and failure of composite structures.

Commercial Finite element method (FEM) software can perform analysis with many types of material nonlinearities, such as plasticity, hyperelasticity, viscoplasticity, etc. However, almost no software contains a module for damage analysis of composite materials.

The goal of this paper is to present the numerical results of elastic damage of thin composite plates. The analysis was performed by user own software, created in MATLAB programming language. This software can perform numerical analysis of elastic damage using finite element method using layered plate finite elements based on the Kirchhoff plate theory. Locking effect was not removed, since this is a rather complicated issue [1]. This paper is organized in three sections: first, a general description of damage is provided, then the damage model used is examined. Finally, some numerical results obtained for damage of plates are presented.

3. THEORY AND NUMERICAL MODELING BACKROUND

A number of material modeling strategies exist to predict failure in laminated composites, subject to severe static or impulsive loads. Broadly, they can be classified as [10,13,16] :

- failure criteria approach (which can be based on the equivalent stress or strain),
- fracture mechanics approach (based on energy release rates),
- plasticity or yield surface approach,
- damage mechanics approach

Strength-based failure criteria are commonly used with the FEM to predict failure events in composite structures. Numerous classical criteria have been derived to relate internal stresses and experimental measures of material strength to the onset of failure (maximum stress or strain, Hill, Hoffman, Tsai-Wu). These classical criteria implemented in most commercial FE codes are not able to physically capture the failure mode. Some of them cannot deal with materials having a different strength in tension and compression. The Hashin criteria are briefly reviewed in [9] and improvements are proposed by Puck and Schurmann [12] over Hashin's theories are examined.

However, few criteria can represent several relevant aspects of the failure process of laminated composites, e.g. the increase on apparent shear strength when applying moderate values of transverse compression, or detrimental effect of the in-plane shear stresses in failure by fiber kinking.

2.1 Damage mechanics

We consider a volume of material free of damage if no cracks or cavities can be observed at the microscopic scale. The opposite state is the fracture of the volume element. Theory of damage describes the phenomena between the virgin state of material and the macroscopic onset of crack [6,14]. The volume element must be of sufficiently large size compared to the inhomogenities of the composite material. In fig. 1 this volume is depicted. One section of this element is related to its normal and to its area *S*. Due to the presence of defects, an effective area for resistance can be found. Total area of defects, therefore, is:

$$S_D = S - \tilde{S} \tag{1}$$

The local damage related to the direction **n** is defined as:

$$D = \frac{S_D}{S} \tag{2}$$

For isotropic damage, the dependence on the normal n can be neglected, i.e.

$$D = D_n \ \forall n \tag{3}$$

We note that damage *D* is a scalar assuming values between 0 and 1. For $D = 0$ the material is undamaged, for $0<D<1$ the material is damaged, for $D = 1$ complete failure occurs. The quantitative evaluation of damage is not a trivial issue, it must be linked to a variable that is able to characterize the phenomenon. Several papers can be found in literature where the constitutive equations of the materials are a function of a scalar variable of damage [2, 3]. For the formulation of a general multidimensional damage model it is necessary to generalize the scalar damage variables. It is therefore necessary to define corresponding tensorial damage variables that can be used in general states of deformation and damage [14].

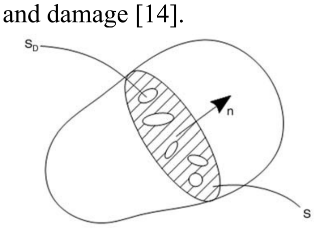

Figure. 1 Representative volume element for damage mechanics.

2.2 Numerical modeling

In composite laminates, defects tend to accumulate at the interface between plies or in intralaminar pockets that are rich in resin. In real structures, both mechanisms of failure are usually present. With respect to FEM simulation, a smeared approach is usually adopted for the simulation of matrix cracking, while delamination is modeled discretely [15]. One of the most powerful computational methods for structural analysis of composites is the FEM. The starting point would be a "validated" FE model, with a reasonably fine mesh, correct boundary conditions, material properties, etc. [1]. As a minimum requirement, the model is expected to produce stress and strains that have reasonable accuracy to those of the real structure prior to failure initiation. In spite of the great success of the finite and boundary element methods as effective numerical tools for the solution of boundary-value problems on complex domains, there is still a growing interest in the development of new advanced methods. Many meshless formulations are becoming popular due to their high adaptivity and a low cost to prepare input data for numerical analysis [5,11,4].

3. USED DAMAGE MODEL

The model for fiber-reinforced lamina mentioned next was presented by Barbero and de Vivo [2] and is suitable for fiber - reinforced composite material with polymer matrix. On the lamina level these composites are considered as ideal homogenous and transversely isotropic. All parameters of this model can be easily identified from available experimental data. It is assumed that damage in principal directions is identical with the principal material directions throughout the damage process. Therefore the evolution of damage is solved in the lamina coordinate system. The model predicts the evolution of damage and its effect on stiffness and subsequent redistribution of stress.

3.1 DAMAGE SURFACE AND DAMAGE POTENTIAL

Damage surface is defined by tensors **J** and **H** [3]

$$J = \begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ 0 & 0 & J_{33} \end{bmatrix}, \quad H = \lfloor H_1 \quad H_2 \quad H_3 \rfloor \tag{4}$$

Damage surface is similar to the Tsai-Wu damage surface [6], which is commonly used for predicting failure of fiber-reinforced lamina with respect to experimental material strength values. Damage surface and damage potential have the form of [3]

$$g(\mathbf{Y},\gamma) = \sqrt{J_{11}Y_1^2 + J_{22}Y_2^2 + J_{33}Y_3^2} + \sqrt{H_1Y_1^2 + H_2Y_2^2 + H_3Y_3^2} - (\gamma + \gamma_0) \tag{5}$$

$$f(\mathbf{Y},\gamma) = \sqrt{J_{11}Y_1^2 + J_{22}Y_2^2 + J_{33}Y_3^2} - (\gamma + \gamma_0) \tag{6}$$

where the thermodynamic forces Y_1, Y_2 and Y_3 can be calculated by relations

$$\begin{aligned} Y_1 &= \frac{1}{\Omega_1^2}\left(\frac{\bar{S}_{11}}{\Omega_1^4}\sigma_1^2 + \frac{\bar{S}_{12}}{\Omega_1^2\Omega_2^2}\sigma_1\sigma_2 + \frac{\bar{S}_{66}}{\Omega_1^2\Omega_2^2}\sigma_6^2 \right) \\ Y_2 &= \frac{1}{\Omega_2^2}\left(\frac{\bar{S}_{22}}{\Omega_2^4}\sigma_2^2 + \frac{\bar{S}_{12}}{\Omega_1^2\Omega_2^2}\sigma_1\sigma_2 + \frac{\bar{S}_{66}}{\Omega_1^2\Omega_2^2}\sigma_6^2 \right) \\ Y_3 &= 0 \end{aligned} \tag{7}$$

where stresses and components of matrix $\bar{S}$ are defined in the lamina coordinate system. Matrix $\bar{S}$ gives the strain-stress relations in the effective configuration [2].

Equation (5) and (6) can be written for different simple stress states: tension and compression in fiber direction, tension in a transverse direction, in-plane shear. Tensors **J** and **H** can be derived in terms of material strength values.

3.2 Hardening parameters

In the present model for damage isotropic hardening is considered and hardening function was used in the form of

$$\gamma = c_1 \left[\exp\left(\frac{\delta}{c_2} \right) + 1 \right] \tag{8}$$

The hardening parameters γ_0, c_1 and c_2 are determined by approximating the experimental stress-strain curves for in-plane shear loading. If this curve is not available, we can reconstruct it using function

$$\sigma_6 = F_6 \tanh\left(\frac{G_{12}}{F_6}\gamma_6 \right) \tag{9}$$

where F_6 is in-plane shear strength, G_{12} is the in-plane initial elasticity modulus and γ_6 is the in-plane shear strain (in the lamina coordinate system). This function represents experimental data very well.

3.3 Critical damage level

The reaching of critical damage level is dependent on stresses in lamina. If in a point in lamina only normal stress occurs in the fiber direction and across the fibers (i.e., normal stress in lamina coordinate system), then simply comparing the values of damaged variables with critical values of damage variables for given material at this point is sufficient. The damage has reached critical level if at least one of the values D_1, D_2 in the point of lamina is greater or equal to the critical value. If in given point of lamina shear stress occurs (in lamina coordinate system), it is additionally necessary to compare the value of the product of $(1 - D_1)(1 - D_2)$ with k_s value from tab. 3 for given material. If the value of this product is less or equal to k_s value, the damage has reached a critical level.

3.4 Implementation of numerical method

Newton-Raphson method was used for solving the system of nonlinear equations. Evolution of damage has been solved using return-mapping algorithm described in [2]. The input values are strains, strain increments in lamina coordinate system, state variables D_1, D_2, and δ in integration point from the start of last performed iteration, $\overline{C}$ matrix (gives the stress-strain relations in effective configuration [3]) and damage parameters related to damage model. The output variables are D_1, D_2, and δ, stresses and strains in lamina coordinate system in this integration point at the end of the last performed iteration. Another output is constitutive damage matrix $\mathbf{C_{ED}}$ in lamina coordinate system, which reflects the effect of damage on the behaviour of structure. Flowchart of numerical damage analysis is described in Fig. 2.

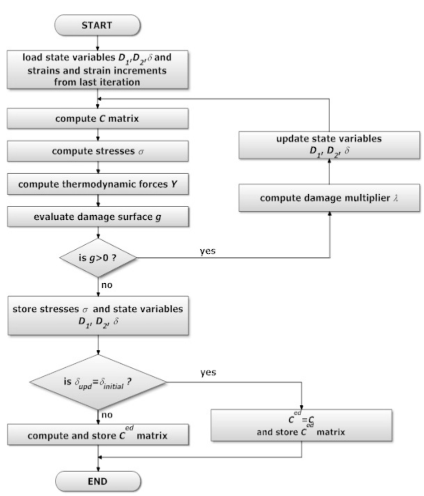

Figure. 2 Flowchart of numerical damage analysis of thin plates.

3. NUMERICAL EXAMPLE

One problem for two different materials were simulated in order to study the damage of laminated fiber reinforced composite plates. The composites are reinforced by carbon fibers embedded in epoxy matrix. The simply supported composite plate with laminate stacking sequence $[0, 45, -45, 90]_S$ with dimensions 125×125×2.5 mm was loaded by transverse force F = - 4000 N in the middle of the plate. Own program created in MATLAB language was used for this analysis.

Material properties, damage parameters and hardening parameters and critical damag**e** values are given in Tabs. 1 - 3. The parameters J_{33} and H_3 are equal to zero. The plate model was divided into 8×8 elements and was analyzed in fifty load substeps. The linear static analysis shows that the largest stress in absolute values are in parallel direction with fibers and transverse to fibers and they occur in the outer layers in the middle of the plate.

Table. 1 Material properties

	$\mathbf{E_1}$[GPa]	$\mathbf{E_2}$[GPa]	$\mathbf{G_{12}}$[GPa]	$\boldsymbol{\upsilon_{12}}$
M30/949	167	8.13	4.41	0.27
M40/948	228	7.99	4.97	0.292

Table. 2 Damage and hardening parameters

	$\mathbf{J_1}$	$\mathbf{J_2}$	$\mathbf{H_1}$
M30/949	$0.952.10^{-3}$	0.438	$25.585.10^{-3}$
M40/948	$2.208.10^{-3}$	0.214	$10.503.10^{-3}$
	$\mathbf{H_2}$	$\boldsymbol{\gamma_0}$	c_1
M30/949	$21.665.10^{-3}$	0.6	0.30
M40/948	$8.130.10^{-3}$	0.12	0.10

Table. 3 Critical values of damage variables

	$\mathbf{D_{1t}^{cr}}$	$\mathbf{D_{1c}^{cr}}$	$\mathbf{D_{2c}^{cr}}$	$\mathbf{k}_s$
M30/949	0.105	0.111	0.5	0.944
M40/948	0.105	0.111	0.5	0.908

The largest shear stresses in absolute value are in the outer layers in corner nodes. The largest absolute stress values are in layers 2, 3, 6 and 7 in the middle of the plate. According to the results of linear static analysis can be expected that damage reaches the critical level in some of the above points. Figs. 3 - 4 show the results of analysis of elastic damage for material M30/949. Fig. 3 shows the evolution of individual stress components in dependence on strains in lamina coordinate system in the midsurface of layer 1 (first layer from bottom) in integration point (IP) 1 (in element 1, nearest to the corner). Fig. 4 shows the evolution of individual stress components in the midsurface of layer 2 in IP 872 (in element 28, nearest to middle of plate). Figs. 5 - 6 plot described damage variables evolution in IPs.

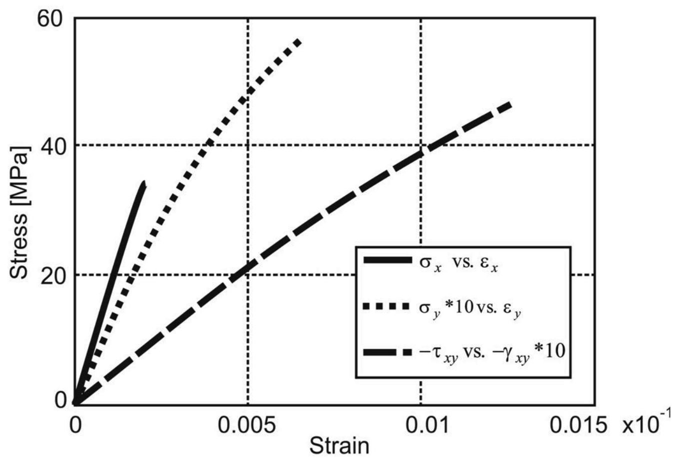

Figure. 3 Stress and strain evolution for material M30/949 in IP 1.

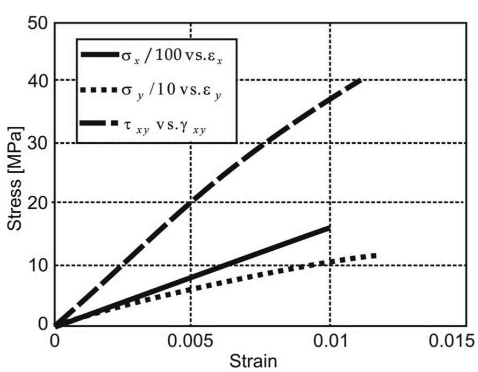

Figure. 4 Stress and strain evolution for material M30/949 in IP 872.

The analysis results show that reaching the critical level is caused not by normal stresses in the lamina coordinate system, but by shear stresses (in the lamina coordinate system). The results of analysis of plate made from material M30/949 show that for a given load the critical level of damage is reached in layers 2 and 7 in the middle of plate and its vicinity. In IPs that are closest to the center of plate in these layers, the critical level of damage was reached between 13th and 14th load substep. However, the model used for the damage does not correctly predicted failures [2]. In some cases, failures occur before the damage reaches a critical level. For given material load F = -1096 N is already critical.

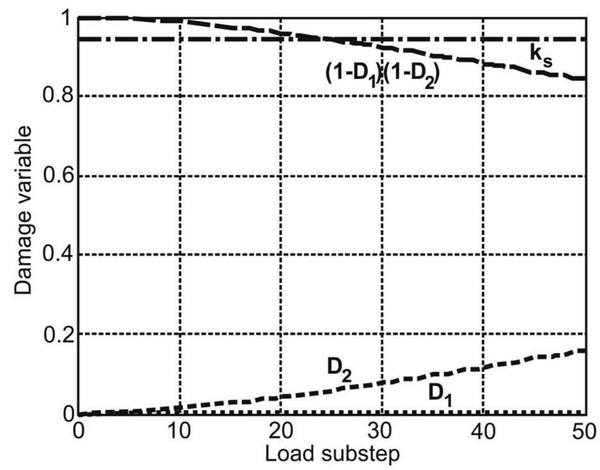

Figure. 5 Damage variables evolution for material M30/949 in IP 1.

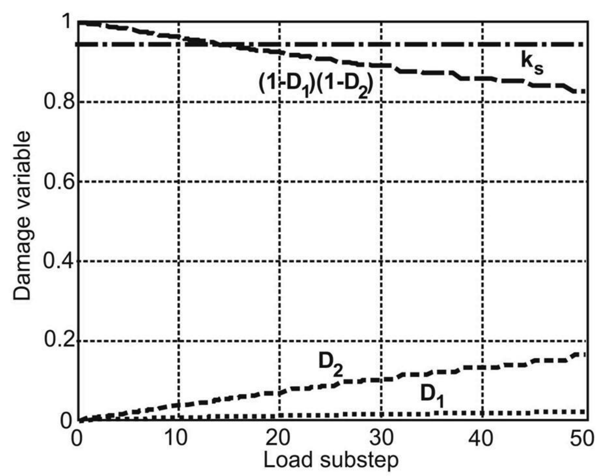

Figure. 6 Damage variables evolution for material M30/949 in IP 872.

Figs. 7 - 8 show the results of analysis of elastic damage for material M40/948. The results show that reaching the critical level of damage will be also caused by shear stresses in lamina coordinate system. However, the critical damage level was reached in the middle of plate and its vicinity in layers 1 and 8. The critical level of damage was reached between 12th and 13th load substep in nearest IPs.

The critical level of damage would be reached in the middle of the plate and its vicinity in layers 2 and 7 (in nearest IPs between 16th and 17th load substep) and also in layers 3 and 6 (in nearest IPs between 27th and 28th load substep). Figs. 9 - 10 show damage variables evolution in IPs. For given material, load F = - 990 N is already critical.

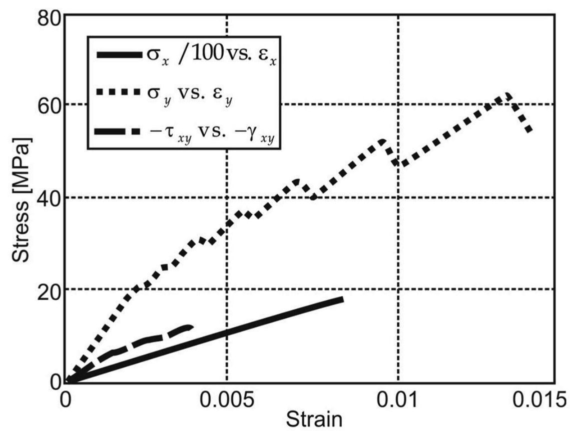

Figure. 7 Stress and strain evolution for material M40/948 in IP 868.

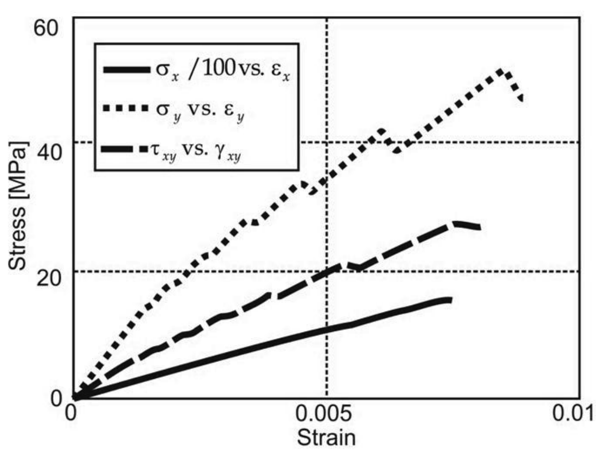

Figure. 8 Stress and strain evolution for material M40/948 in IP 872.

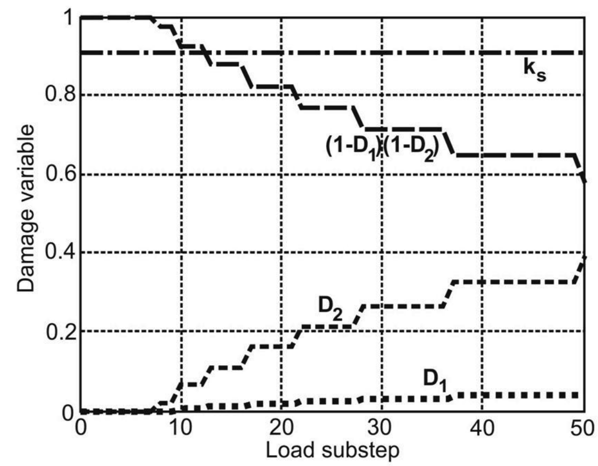

Figure. 9 Damage variables evolution for material M40/948 in IP 868.

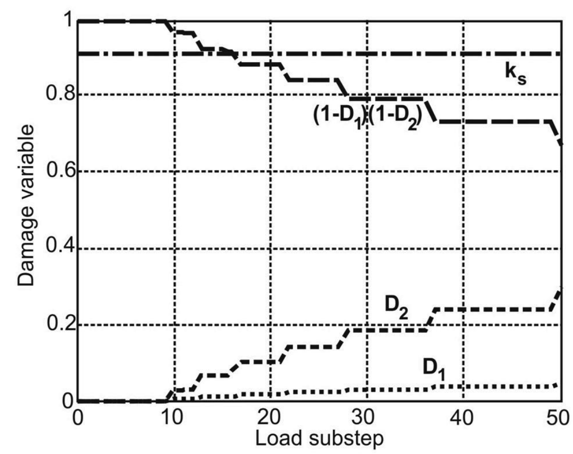

Figure. 10 Damage variables evolution for material M40/948 in IP 872

3. CONCLUSION

In the current study, we focused on programming the damage of thin unidirectional fiber-reinforced composite plates. The postulated damage surface reduces to the Tsai-Wu surface in stress space. However, presented model goes far beyond simple failure criteria by identifying a damage threshold, hardening parameters for the evolution of damage, and the critical values of damage for which material failure occurs. The analysis results show that the change of the material has a significant influence on the evolution of damage as well as on location of critical damage and load at which critical level of damage is reached. For material M30/949 damage reached critical level in the middle of the plate and its vicinity in layers 2 and 7, and for material M40/948 damage has reached a critical level also in the middle of the plate and its vicinity, but in layers 1 and 8.

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