ZESZYTY NAUKOWE WYŻSZEJ SZKOŁY PEDAGOGICZNEJ w BYDGOSZCZY
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# SOME REMARKS ABOUT CEDER-LEVI THEOREM

Let $X$ and $Y$ be topological spaces. Recall that a multifunction $F: X \to Y$ is said to be of lower class $\alpha$ if $F^{-}(U) := \{x: F(x) \cap U = \emptyset\}$ is a Borel set in $X$ of additive class $\alpha$ for each open set $U$ in $Y$. Interrelations between above notion and usual Baire classification were investigated in [3].

In [1] (see also [2]) is inductively proved, that the multifunction of lower class $\alpha$ with convex but not necessarily closed values in finite dimensional linear space possesse a Borel $\alpha$ selector. We may ask whether the range space in Ceder-Levi theorem can be generalized. Some negative results in this direction were given in [8]. This note is devoted to positive one. Namely, we exhibite this fact, that the Michael's [6] methods gives more powerful theorem on Borel $\alpha$ selectors.

Let $Y$ be a Banach space or, more generally a Frechet space. If $K$ is a close, convex subset of $Y$, then a supporting set of $K$ is, by definition, a closed, convex proper subset $S$ of $K$ (in particular a singleton), such that if an interior point of a segment in $K$ belongs to $S$, then the whole segment is contained in $S$. The set $I(K)$ of all elements of $K$ which are not in any supporting set of $K$ will be called the inside of $K$. The family $D(Y) = \{B \subset Y : B = \text{conv } B \text{ and } B \supset I(\text{Cl } B)\}$ is seemingly the adequate range space for the Ceder-Levi theorem. Emphasize that every convex set which is either closed, or has an interior point, or is finite deimensional, belongs to $D(Y)$ (see [6], p.372). We prove the following improvement of the Ceder-Levi theorem:

THEOREM. Let $F: X \to D(Y)$ where $X$ is a perfectly normal topological space and $Y$ is a separable Frechet space. If $F$ is of lower class $\alpha > 0$, then $F$ has a Borel selector.

Proof. Define $\check{F}: X \to Y$ by formula $\check{F}(x) = \mathrm{Cl}\, F(x)$ ; what we must find is a Borel $\alpha$ selector $f: X \to Y$ such that $f(x) \in I(\check{F}(x))$ for every $x \in X$. Obviously $\check{F}$ is also of lower class $\alpha$. Thus, in virtue of the theorem 4 of [5], $\check{F}$ has the so-called Castaing's representation. Namely, there exist functions $f_i: X \to Y$, $i=1,2,\ldots$, such that each $f_i$ is a Borel $\alpha$ function and we have the equality

$$\check{F}(x) = \mathrm{Cl}\ (\{f_i(\ ): i=1,2,\ldots\})$$

on the whole X.

Now, let $g_i(x) = f_1(x) + \dfrac{f_i(x) - f_1(x)}{\max(1,\ d(f_i(x),\ f_1(x)))}$

$i=1,2,\ldots$, where $d$ denote the invariant metric on $Y$.

Put $f(x) = \sum_{i=1}^{\infty} 2^{-i} g_i(x)$ .

An inspection of the proof of the lemma 5.1 in [6] shows that $f(x) \in I(\check{F}(x)) \subset F(x)$ for every $x \in X$. Since the series defining $f: X \to Y$ converges almost uniformly on $X$, it follows that $f$ is also of Borel class $\alpha$, and thus has all the required properties.

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