# Total to selective extinction ratio $R=A_{V} / E(B-V)$ in Cassiopeia field Walter Wegner 

The four fields have been selected at the Astronomical Observatory of N. Copernicus University in Toruń about 30 years ago in order to investigate the structure of Milky Way. One of the fields is situated in the Cassiopeia region centered at $\alpha(1950)=23^{h} 57^{m}, \delta(1950)=$ $+59^{\circ} .6\left(l_{I I}=+84^{\circ}, b_{I I}=-2^{\circ}\right)$ and covering the area of about 18.1 square degrees. In this area the author (1989) calculated the ratio of total-to-selective extinction $R=A_{V} / E(B-V)$ from star counts. The result received suggests, that the value of $R$ varied from place to place and very strongly depended on distance to stars. For distant stars the author received $R=3.96 \pm 0.12$ and this result is higher than the value $R$ cited by C. Schalén (1975). He suggests that the value of $R$ for stars in this region and other regions of the sky is 3.1 and variations of $R$ seem to be small. In this paper the author tested the value of $R$ for this region applying different methods based on stellar photometry available in different spectral bands from near $I R$ to near $U V$ (methods 2 and 3 ), over optical range (method 4) to $U V$ band (method 1). The resulting $R$ values differ strongly and the mean value is $R=3.58 \pm 0.09$ consistent with that obtained from star counts in the limits of their large mean errors.

## 1. Introduction

In Stars and Stellar Systems, Vol. VII, p. 167, H.L. Johnson has written an article on interstellar extinction (Johnson 1968). The main result of this paper is that the quantity $R=A_{V} / E(B-V)$ i.e. the ratio of total visual extinction $A_{V}$ to colour excess $E(B-V)$ varies from a value of about 3 in some regions to values as high as 6 or 7 in other regions. The authors Wegner (1986, 1987, 1988, 1989), Krełowski and Wegner (1989), Krełowski, Papaj, Wegner (1990), Wegner, Papaj and Krelowski (1990) and many other authors suggested that the value of $R$ differs from place to place and from stars to stars. If these results prove to be true they are of the greatest importance for all investigations on the structure of stellar systems.

## 2. The material

In the investigated field in Cassiopeia ( $23^{h} 40^{m}-0^{h} 05^{m} ; 57^{\circ}-62^{\circ} 10^{\prime}$ ) there are 20 stars (see Table 1) for which the photometric data for near $I R$, optical $V, B, U$ and far $U V$ are known. Table 1 contains: the number of star, $\alpha(1950), \delta(1950), S p / L, V, B-V, U-B$ and $E(B-V)$. The $I R$ photometric magnitudes are taken from Gezari et al. (1984), the $U, B, V$ and the $U V$ magnitudes from ANS CatalogueWesselius et al. (1982).
For stars brighter than $13^{m}$ the photographic $m_{p g}$ and photovisual $m_{p v}$ magnitudes are taken from catalogues of Hutorowicz (1956) and Ampel (1959). These magnitudes are transformed to $B, V$ system using the transformation formulae established by Wegner (1978) assuming the photometric zero points established on the basis of photoelectric measurements available in this field for 60 stars ( $\left.23^{h} 40^{m}-0^{h} 04^{m} ; 57^{\circ}-61^{\circ}\right)$. Altogether $B, V$ and $S p / L$ are derived (with the aid of the $24^{\prime \prime} / 36^{\prime \prime}$ Schmidt-Cassegrain telescope and the $5^{\circ}$ flint objective prism) for 1103 stars.

Table 1.
Primary data for reddened stars in Cassiopeia

| No | Name* | $\alpha$ (1950) | $\delta(1950)$ | $S p / L$ | $V$ | $B-V$ | $U-B$ | $E(B-V)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $61^{\circ} 2509$ | $23^{h} 41^{m} 23^{s}$ | $61^{\circ} 53^{\prime} .2$ | B0.5Ib | $8^{m} 42$ | $0^{m} 46$ | $-0^{m} 55$ | $0^{m} 67$ |
| 2 | 602615 | 234217 | 6123.4 | B0.5Ib | 9.10 | 0.60 | -0.45 | 0.81 |
| 3 | 612515 | 234317 | 6159.9 | B0.5V | 9.96 | 0.43 | -0.51 | 0.72 |
| 4 | 612526 | 234515 | 6146.2 | B2Ib | 8.77 | 0.39 | -0.50 | 0.55 |
| 5 | 612529 | 234603 | 6142.7 | B1Ib | 8.66 | 0.54 | -0.46 | 0.73 |
| 6 | 223385 | 234623 | 6156.2 | A3Iae | 5.43 | 0.67 | -0.02 | 0.61 |
| 7 | 612550 | 234951 | 6150.4 | O9.5II | 9.29 | 0.32 | -0.63 | 0.61 |
| 8 | 612559 | 235111 | 6209.1 | 09V | 9.72 | 0.29 | -0.66 | 0.60 |
| 9 | 223987 | 235143 | 6119.7 | B1Ib | 7.56 | 0.50 | -0.49 | 0.69 |
| 10 | 224055 | 235212 | 6133.6 | B3Iae | 7.17 | 0.70 | -0.22 | 0.83 |
| 11 | 224151 | 235303 | 5708.0 | B0.5II | 6.00 | 0.21 | -0.72 | 0.48 |
| 12 | 223960 | 235120 | 6034.5 | A0Ia | 6.90 | 0.71 | -0.05 | 0.70 |
| 13 | 224424 | 235516 | 5926.5 | B1Iab | 8.10 | 0.75 | -0.21 | 0.94 |
| 14 | 240464 | 235520 | 5920.0 | O9V | 9.59 | 0.31 | -0.61 | 0.62 |
| 15 | 224599 | 235642 | 5944.7 | B0.5Ve | 9.56 | 0.42 | -0.52 | 0.71 |
| 16 | 224905 | 235905 | 6010.3 | $B 1 V e$ | 8.47 | 0.14 | -0.56 | 0.41 |
| 17 | 225146 | 00122 | 6049.5 | BoIb | 8.60 | 0.37 | -0.64 | 0.60 |
| 18 | 225160 | 00128 | 6156.6 | O8e | 8.19 | 0.26 | -0.72 | 0.57 |
| 19 | 602668 | 00325 | 6035.9 | B1III | 8.95 | 0.46 | -0.48 | 0.73 |
| 20 | 592829 | 00409 | 6020.6 | BoIV. | 9.84 | 0.40 | -0.65 | 0.70 |
| ( $B D$ ’or $H D$ ) |  |  |  |  |  |  |  |  |

## 3. The method

Using the pair method ( $m_{\text {reddened }}^{\text {star }}-m_{\text {unreddened }}^{\text {standard }}$ ), where the spectral and luminosity clasification of standard star and reddened star are the same, were calculated for every star presented in Table 1 and the extinction in the form $E(\lambda-V) / E(B-V)$ versus $1 / \lambda$ was derived. The normalization corresponds to $A_{V}=0$ and $A_{B}=1$. The $V, B, S p$ and $L$ data for reddened and standard stars are known unprecisely, therefore the choice of standard star is a very important problem. At the best we may calculate the extinction using some natural standards (see Papaj, Wegner, Krełowski 1990) or the "artificial standards" (in preparation). Figure 1
presents the typical extinction curve as relation $E(\lambda-V) / E(B-V)$ versus $1 / \lambda$ for reddened star $H D 144217$ (according to Bright Star Catalogue $\left.\beta^{1} S c o, B 1 V M K, E(B-V)=0^{m} .17\right)$. The photometric data are taken from TD-1 Atlas spectra (Jamar et al. 1976, Macau-Hercot et al. 1978). This high quality spectrum was divided by "artificial standards" $B 0 V$ and $B 1 V$ spectral types. The strong spectral feature of $C I V$ (between $1 / \lambda=6$ and $1 / \lambda=7$ ) is apparently observed either in absorption or in emission in the two curves. The mean curve does not contain any remnant of this feature and, in fact, the $\beta^{1}$ Sco is classified as $B 0.5 \mathrm{~V}$ see Papaj, Wegner, Krelowski (1991). Let us mention also that, when the spectral type of the standard is later (see Figure 2), a small depression between the normalization point ( $2740 \hat{A}$ ) and the next one ( $2540 \AA$ ) may be created. This feature, when present, may be considered as the result of mismatch because it is evidently hard to find any physical reason for the "blueing" of the star under consideration right after the point of normalization. In this paper we applied the natural standards. List of these natural standard stars are presented in the Table 2. Now we may compare a calculation of extinction obtained with the aid of natural standard stars or "artificial standard" stars for $B 1 V$ stars only (Wegner, Papaj, Krełowski - in preparation). These comparisons are presented in Table 3 for three stars: No. 3 ( $B 0.5 \mathrm{~V}$ ), No. 15 ( $B 0.5 \mathrm{Ve}$ ) and No. 16 ( $B 1 \mathrm{Ve}$ ).
The values of $R=A_{V} / E(B-V)$ were calculated with the aid of four different methods.

Method 1.
In the paper by Cardelli et al. (1982) a relation between $A_{\lambda} / A_{V}$ and $R$ is derived:

$$
\begin{equation*}
\frac{A_{\lambda}}{A_{V}}=a(x)+b(x) / R \tag{1}
\end{equation*}
$$

The authors cited the expression formulae $a(x), b(x)$ where $x=1 / \lambda$ and $0.12 \leq \lambda \leq 1.25[\mu \mathrm{~m}]$. This relation is presented in Figure 3 for three stars: $B D+56^{\circ} 524(R=2.75), H D 154445(R=3.61)$ and Her 36 ( $R=5.3$ ).

Method 2.
In the range $0.25 \leq \lambda \leq 3.6$ [ $\mu \mathrm{m}$ ], the extinction curve may be extrapolated by the best straight line. The point where the extrapolated

Table 2.
Primary data for standard stars

| No | HD | $S p / L$ | $\checkmark$ | $B-V$ | $U-B$ | $E(B-V)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 47839 | O8IIIf | $4^{m} 65$ | $-0^{m} .25$ | $-1^{m} .06$ | $0^{m} .06$ |
| 2 | 214680 | O8III | 4.88 | 0.20 | -1.04 | 0.11 |
| 3 | 14633 | 08/9V | 7.46 | -0.21 | - | 0.10 |
| 4 | 37043 | O9III | 2.77 | -0.26 | -1.06 | 0.05 |
| 5 | 57682 | O9V | 6.43 | -0.19 | -1.04 | 0.12 |
| 6 | 36486 | O9.5I | 2.23 | -0.22 | -1.05 | 0.05 |
| 7 | 37742 | O9.5Ibe | 1.77 | -0.21 | -1.07 | 0.06 |
| 8 | 37468 | O9.V | 3.80 | -0.24 | -1.01 | 0.06 |
| 9 | 38666 | 09.5V | 5.17 | -0.28 | -1.06 | 0.02 |
| 10 | 37128 | BoIae | 1.70 | -0.19 | -1.02 | 0.05 |
| 11 | 63922 | B0III | 4.11 | -0.18 | -1.01 | 0.12 |
| 12 | 36512 | BoV | 4.62 | -0.26 | -1.07 | 0.04 |
| 13 | 38771 | B0.5Ia | 2.06 | -0.17 | -1.03 | 0.05 |
| 14 | 34816 | B0.5IV | 4.29 | -0.26 | -1.03 | 0.01 |
| 15 | 93030 | $B 0.5 \mathrm{~V} p$ | 2.76 | -0.22 | -1.00 | 0.06 |
| 16 | 91316 | $B 1 I a b$ | 3.85 | -0.14 | -0.97 | 0.05 |
| 17 | 214080 | B1Ib | 6.80 | -0.14 | -0.92 | 0.05 |
| 18 | 44743 | B1II-III | 1.98 | -0.25 | 0.97 | 0.03 |
| 19 | 50507 | B1III | 4.83 | -0.21 | -0.96 | 0.05 |
| 20 | 116658 | $B 1 I I I-I V+B 2 V$ | 0.98 | -0.23 | -0.93 | 0.03 |
| 21 | 68324 | B1IVe | 5.24 | -0.22 | -0.89 | 0.04 |
| 22 | 31726 | $B 1 V$ | 6.15 | -0.21 | - | 0.04 |
| 23 | 35715 | $B 1 V$ | 4.59 | -0.22 | -0.90 | 0.04 |
| 24 | 37018 | $B 1 V$ | 4.58 | -0.21 | -0.91 | 0.05 |
| 25 | 127972 | B1.5Vne | 2.31 | -0.19 | -0.82 | 0.06 |
| 26 | 165024 | B2Ib | 3.66 | -0.09 | -0.85 | 0.07 |
| 27 | 52089 | B2II | 1.50 | -0.21 | -0.93 | 0.00 |
| 28 | 51283 | B2II-III | 5.30 | -0.18 | -0.80 | 0.03 |
| 29 | 35468 | B2III | 1.64 | -0.22 | -0.88 | 0.02 |
| 30 | 53138 | B3Ia | 3.01 | -0.08 | -0.80 | 0.05 |
| 31 | 202850 | B9Iab | 4.23 | 0.12 | -0.39 | 0.10 |
| 32 | 212593 | B9Iab | 4.58 | 0.09 | -0.34 | 0.09 |
| 33 | 176437 | B9III | 3.24 | -0.05 | -0.08 | 0.03 |
| 34 | 186882 | B9.51II | 2.87 | -0.02 | -0.10 | 0.03 |


| No | $H D$ | $S p / L$ | $V$ | $B-V$ | $U-B$ | $E(B-V)$ |
| ---: | ---: | :--- | :---: | :---: | :---: | :---: |
| 35 | 218045 | $B 9.5 I I I$ | 2.48 | -0.04 | -0.10 | 0.04 |
| 36 | 46300 | $A 0 I b$ | 4.49 | 0.02 | -0.28 | 0.02 |
| 37 | 87737 | $A 0 I b$ | 3.58 | -0.01 | -0.24 | -0.01 |
| 38 | 167356 | $A 0 I a$ | 6.07 | 0.20 | - | 0.20 |
| 39 | 77350 | $A 0 I I I$ | 5.43 | -0.05 | -0.12 | -0.02 |
| 40 | 110304 | $A 0 I I I$ | 2.16 | -0.01 | -0.01 | 0.02 |
| 41 | 123299 | $A 0 I I I$ | 3.65 | -0.05 | -0.08 | -0.02 |
| 42 | 5550 | $A 0 I I I$ | 5.97 | -0.02 | -0.13 | 0.01 |
| 43 | 197345 | $A 2 I a e$ | 1.25 | 0.09 | -0.23 | 0.04 |
| 44 | 176687 | $A 2 I I I+A 4 I V$ | 2.60 | 0.08 | 0.08 | 0.03 |
| 45 | 102878 | $A 3 I a b$ | 5.70 | 0.26 | -0.04 | 0.21 |
| 46 | 103516 | $A 3 I b$ | 5.91 | 0.19 | 0.01 | 0.13 |
| 47 | 104035 | $A 3 I b$ | 5.61 | 0.18 | -0.11 | 0.12 |
| 48 | 125835 | $A 3 I b$ | 5.61 | 0.49 | 0.01 | 0.43 |

curve intersects the ordinate axis is by definition equal to $-R$.
Method 3.
The $R$ value may be calculated by formula cited by Aiello et al. (1987)

$$
\begin{equation*}
R=1.1 * E(V-K) / E(B-V) \tag{2}
\end{equation*}
$$

Method 4.
In this method it is supposed, that the difference between mean $m_{B}$ and mean $m_{V}$ magnitudes of the neighbouring fields of the sky indicate the absorption $A_{B}$ and $A_{V}$ and hence

$$
R=A_{V} /\left(A_{B}-A_{V}\right)
$$

The details of this method are discussed in papers by Wegner (1986, 1987, 1988). In this method the mean error of the value of $R$ is about $10 \%$ if the investigated field was divided into several hundred smaller field in which the mean $V$ and $B$ magnitudes were calculated for the same stars, and the number of stars in each smaller field is about over a dozen or so. In this paper the field $23^{h} 40^{m} \leq \alpha \leq 0^{h} 04^{m} ; 58^{\circ} \leq$ $\delta \leq 61^{\circ}$ was divided into 162 smaller fields $\left(20^{\prime} \times 20^{\prime}\right)$ in which the average number of stars was about 7 (together 1103 stars were used in calculation for which $V, B$ and $S p / L$ data are assigned). Figure 4 shows the relation between $\Delta A_{V}$ and $\Delta A_{B}$ in this area.

Table 3.
The extinction $E(\lambda-V) / E(B-V)$ with mean errors versus $1 / \lambda$ for 20 reddened stars in Cassiopeia

| star | $E(\lambda-V) / E(B-V)$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{lllll}\text { star } & 3.5 & 2.2 & 1.62 & 1.25\end{array}$ | $V$ | $B$ | $U$ | 0.33 | 0.25 | 0.22 | 0.18 | 0.15 |
| 1 | $\begin{gathered} 6,7,10,11,13,14 \\ 16,17,19 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.70 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 1.85 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 3.83 \\ & 0.10 \end{aligned}$ | $\begin{aligned} & 6.38 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 4.49 \\ & 0.40 \end{aligned}$ | $\begin{aligned} & 4.93 \\ & 0.43 \end{aligned}$ |
| 2 | $\begin{gathered} 6,7,10,11,13,14 \\ 16,17,19 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.70 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 1.98 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 3.99 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 6.98 \\ & 0.24 \end{aligned}$ | $\begin{aligned} & 4.65 \\ & 0.32 \end{aligned}$ | $\begin{aligned} & 5.05 \\ & 0.35 \end{aligned}$ |
| 3 | $\begin{gathered} 8,9,12,14,15,20 \\ 21,22,23,24,25 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.61 \\ & 0.19 \end{aligned}$ | $\begin{aligned} & 2.42 \\ & 0.15 \end{aligned}$ | $\begin{aligned} & 4.38 \\ & 0.18 \end{aligned}$ | $\begin{aligned} & 7.09 \\ & 0.09 \end{aligned}$ | $\begin{aligned} & 5.36 \\ & 0.27 \end{aligned}$ | $\begin{aligned} & 6.20 \\ & 0.25 \end{aligned}$ |
|  | artificial standard B1V |  |  |  | 2.41 | 4.35 | 7.05 | 5.27 | 6.09 |
| 4 | $\begin{gathered} 16,17,18,26,27 \\ 29 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.66 \\ & 0.19 \end{aligned}$ | $\begin{aligned} & 1.83 \\ & 0.20 \end{aligned}$ | $\begin{aligned} & 4.07 \\ & 0.09 \end{aligned}$ | $\begin{aligned} & 6.73 \\ & 0.16 \end{aligned}$ | $\begin{aligned} & 4.00 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 3.88 \\ & 0.10 \end{aligned}$ |
| 5 | $\begin{gathered} 6,7,10,11,13,14 \\ 16,17,19 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.74 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 1.91 \\ & 0.07 \end{aligned}$ | $\begin{aligned} & 3.97 \\ & 0.09 \end{aligned}$ | $\begin{aligned} & 6.57 \\ & 0.11 \end{aligned}$ | $\begin{aligned} & 4.69 \\ & 0.34 \end{aligned}$ | $\begin{aligned} & 5.22 \\ & 0.37 \end{aligned}$ |
| 6 | $\begin{array}{r} 43,44,45,46,47,48-2.90-2.77 \\ 0.220 .13 \end{array}$ | 0 | 1 | $\begin{aligned} & 1.05 \\ & 0.09 \end{aligned}$ | $\begin{aligned} & 1.17 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 1.75 \\ & 0.31 \end{aligned}$ | $\begin{aligned} & 4.16 \\ & 0.37 \end{aligned}$ | $\begin{aligned} & 1.89 \\ & 0.39 \end{aligned}$ | $\begin{aligned} & 1.19 \\ & 0.27 \end{aligned}$ |
| 7 | $\begin{gathered} 1,2,4,6,7,10,11 \\ 13 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.77 \\ & 0.02 \end{aligned}$ | $\begin{aligned} & 2.09 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 4.08 \\ & 0.10 \end{aligned}$ | $\begin{aligned} & 6.74 \\ & 0.10 \end{aligned}$ | $\begin{aligned} & 5.51 \\ & 0.69 \end{aligned}$ | $\begin{aligned} & 4.96 \\ & 0.48 \end{aligned}$ |
| 8 | 3,5,8,9,12 | 0 | 1 | $\begin{aligned} & 1.61 \\ & 0.23 \end{aligned}$ | $\begin{aligned} & 2.18 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 4.20 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 6.55 \\ & 0.09 \end{aligned}$ | $\begin{aligned} & 5.09 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 5.52 \\ & 0.14 \end{aligned}$ |
| 9 | $\begin{gathered} 6,7,10,11,13,14 \\ 16,17,19 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.75 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 2.07 \\ & 0.07 \end{aligned}$ | $\begin{aligned} & 4.17 \\ & 0.10 \end{aligned}$ | $\begin{aligned} & 6.65 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 5.02 \\ & 0.35 \end{aligned}$ | $\begin{aligned} & 5.29 \\ & 0.39 \end{aligned}$ |
| 10 | 26,28,30 | 0 | 1 | $\begin{aligned} & 1.73 \\ & 0.07 \end{aligned}$ | $\begin{aligned} & 2.18 \\ & 0.14 \end{aligned}$ | $\begin{aligned} & 4.41 \\ & 0.25 \end{aligned}$ | $\begin{aligned} & 6.86 \\ & 0.21 \end{aligned}$ | $\begin{aligned} & 5.28 \\ & 0.34 \end{aligned}$ | $\begin{aligned} & 5.70 \\ & 0.37 \end{aligned}$ |
| 11 | $\begin{array}{crr} 6,7,10,13,17,18, & -2.74-2.57 \\ 19,26,30 & 0.15 & 0.11 \end{array}$ | 0 | 1 | $\begin{aligned} & 1.52 \\ & 0.04 \end{aligned}$ |  |  |  |  |  |
| 12 | $\begin{array}{rrr} 31,32,33,34,35,36, & -3.10-2.83 \\ 37,38,39,40,41,42 & 0.24 & 0.04 \end{array}$ | 0 | 1 | $\begin{aligned} & 1.20 \\ & 0.07 \end{aligned}$ | $\begin{aligned} & 1.30 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 3.70 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 6.05 \\ & 0.11 \end{aligned}$ | $\begin{aligned} & 4.55 \\ & 0.15 \end{aligned}$ | $\begin{aligned} & 5.33 \\ & 0.17 \end{aligned}$ |
| 13 | $\begin{array}{crrrr} 6,7,10,13,17,18, & -2.72-2.67-2.46 & -2.13 \\ 19,26,30 & 0.11 & 0.08 & 0.07 & 0.05 \end{array}$ | 0 | 1 | $\begin{aligned} & 1.77 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 2.04 \\ & 0.03 \end{aligned}$ | $\begin{aligned} & 3.80 \\ & 0.04 \end{aligned}$ | $\begin{aligned} & 5.69 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 5.12 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 5.30 \\ & 0.10 \end{aligned}$ |
| 14 | - $3,5,8,9,12$ | 0 | 1 | $\begin{aligned} & 1.68 \\ & 0.22 \end{aligned}$ | $\begin{aligned} & 2.12 \\ & 0.05 \end{aligned}$ | $\begin{aligned} & 4.19 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 6.62 \\ & 0.13 \end{aligned}$ | $\begin{aligned} & 4.91 \\ & 0.11 \end{aligned}$ | $\begin{aligned} & 5.21 \\ & 0.11 \end{aligned}$ |
| 15 | $\begin{gathered} 8,9,12,14,15,20 \\ 21,22,23,24,25 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.64 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 2.30 \\ & 0.16 \end{aligned}$ | $\begin{aligned} & 4.37 \\ & 0.18 \end{aligned}$ | $\begin{aligned} & 6.83 \\ & 0.10 \end{aligned}$ | $\begin{aligned} & 5.31 \\ & 0.27 \end{aligned}$ | $\begin{aligned} & 5.88 \\ & 0.27 \end{aligned}$ |
|  | artificial standard $B 1 V$ |  |  |  | 2.28 | 4.34 | 6.78 | 5.22 | 5.78 |
| 16 | 14,15,20,21,22,23, | 0 | 1 | 2.01 | 2.50 | 5.08 | 7.76 | 6.58 | 7.20 |
|  | artificial standard $B 1 V$ |  |  |  | 2.59 | 5.14 | 7.76 | 6.59 | 7.16 |
|  | 24,25 |  |  | 0.15 | 0.21 | 0.29 | 0.14 | 0.39 | 0.37 |
| 17 | $\begin{array}{cr} 6,7,10,13,16,17, & -2.54-2.62 \\ 18,19,26,30 & 0.04 \\ 0.02 \end{array}$ | 0 | 1 | $\begin{aligned} & 1.50 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 1.73 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 3.63 \\ & 0.22 \end{aligned}$ | $\begin{aligned} & 6.65 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 3.72 \\ & 0.40 \end{aligned}$ | $\begin{aligned} & 4.67 \\ & 0.56 \end{aligned}$ |
| 18 | $\begin{array}{cr} 1,2,3,4,5,6,7, & -3.69-3.07 \\ 8,9 & 0.04 \end{array}$ | 0 | 1 | $\begin{aligned} & 1.71 \\ & 0.03 \end{aligned}$ | $\begin{aligned} & 2.01 \\ & 0.01 \end{aligned}$ | $\begin{aligned} & 4.07 \\ & 0.02 \end{aligned}$ | $\begin{aligned} & 6.53 \\ & 0.04 \end{aligned}$ | $\begin{aligned} & 4.44 \\ & 0.02 \end{aligned}$ | $\begin{aligned} & 5.16 \\ & 0.10 \end{aligned}$ |
| 19 | $\begin{gathered} 4,10,11,18,19,28 \\ 29 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 1.68 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 2.01 \\ & 0.21 \end{aligned}$ | $\begin{aligned} & 4.01 \\ & 0.36 \end{aligned}$ | $\begin{aligned} & 6.76 \\ & 0.43 \end{aligned}$ | $\begin{aligned} & 4.77 \\ & 0.65 \end{aligned}$ | $\begin{aligned} & 5.25 \\ & 0.50 \end{aligned}$ |
| 20 | $\begin{gathered} 8,9,12,14,15,20 \\ 21,22,23,24,25 \end{gathered}$ | 0 | 1 | $\begin{aligned} & 2.01 \\ & 0.12 \end{aligned}$ | $\begin{aligned} & 2.50 \\ & 0.15 \end{aligned}$ | $\begin{aligned} & 5.08 \\ & 0.18 \end{aligned}$ | $\begin{aligned} & 7.76 \\ & 0.11 \end{aligned}$ | $\begin{aligned} & 6.58 \\ & 0.28 \end{aligned}$ | $\begin{aligned} & 7.20 \\ & 0.27 \end{aligned}$ |

## 4. The results

In Table 3 are given: the number of the star according to Table 1, the numbers of natural standard stars according to Table 2, the mean values of extinction $E(\lambda-V) / E(B-V)$ and (their mean error) calculated with the aid of natural standard stars in $I R$ bands (3.5, 2.2, 1.62 and $1.25 \mu \mathrm{~m})$ in optical range $V, B, U$ and for far $U V A N S$ bands ( 0.33 , $0.25,0.22,0.18$ and $0.155 \mu \mathrm{~m})$. For example in Table 3 are given too the calculated values of extinction $E(\lambda-V) / E(B-V)$ obtained with the aid of "artificial standard" (spectral type $B 1 V$ - see Papaj, Wegner, Krelowski - in preparation). The agreement of these results with results obtained with the natural standard stars is perfect.
In Table 4 are given the mean values of $R=A_{V} / E(B-V)$ calculated with the aid of methods $1-4$. The mean value obtained with the method 1 amounts to $R_{1}=3.31 \pm 0.13$. This value was calculated with the aid of 5 fitted points ( $0.33 .0 .25,0.22,0.18$ and 0.15 bands).
The mean value of $R$ obtained with the aid of the method 2 amounts to $R_{2}=3.48 \pm 0.15$ if a straight line is drawn through the $I R$ bands, through $V, B, U$ bands and through the 0.33 and 0.25 bands, or $R_{2}=$ $3.39 \pm 0.08$, if the straight line is drawn through $V, B, U, 0.33$ and 0.25 bands (the IR bands are absent).
The coefficient of the linear correlation amounts to $r=0.976 \pm 0.015$ in the first case and $r=0.987 \pm 0.002$ in the second case of the method 2 . The mean value of $R$ obtained with the aid of the method 3 amounts to $R_{3}=3.08 \pm 0.09$, but single results are very uncertain. If we assume, that the accuracy of $V, B$ and $K$ bands is even $0^{m} .02$, that of $R$ is $\Delta R=0.4$.
The mean value of $R$ obtained with the aid of the method 4 amounts to $R_{4}=3.81 \pm 0.18$. This result is very similar to that obtained by method 1 and also to that of the very labour-taking method of star counts (in this region in Cassiopeia for which the author (1989) obtained $R=4.0 \pm 0.2$ ). That means, that with the aid of the method 4 (the method of surface brightness) the value of $R$ may be derived with the error of about 10 percent and the unprecision of this method when the $I R$ and far $U V$ photometric measurements are not numerous, permits only to estimate the value of $R$. With the aid of this method Valentijn (1990) discovered opaque dense clouds by measuring surface

Table 4.
The $R=A_{V} / E(B-V)$ values received with aid of methods discussed in this paper

| star |  | $R$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | methods |  |  |  |  |  |
|  |  | 1 |  | 2 |  | 3 | 4 |
| 1 |  | 4.14 | 0.18 |  | 3.08 |  |  |
| 2 |  | 3.85 | 0.11 |  | 3.25 |  |  |
| 3 |  | 3.49 | 0.18 |  | 3.66 |  |  |
| 4 |  | 4.25 | 0.08 |  | 3.33 |  |  |
| 5 |  | 3.96 | 0.18 |  | 3.22 |  |  |
| 6 |  |  |  | 3.30 | - | 3.32 |  |
| 7 |  | 3.97 | 0.27 |  | 3.34 |  |  |
| 8 |  | 3.95 | 0.25 |  | 3.47 |  |  |
| 9 |  | 4.06 | 0.31 |  | 3.43 |  |  |
| 10 |  | 3.79 | 0.27 |  | 3.67 |  |  |
| 11 |  |  |  |  |  | 2.83 |  |
| 12 |  | 4.18 | 0.30 | 3.51 | 2.98 | 3.11 |  |
| 13 |  | 4.11 | 0.38 | 3.42 | 3.05 | 2.94 |  |
| 14 |  | 4.07 | 0.26 |  | 3.45 |  |  |
| 15 |  | 3.68 | 0.21 |  | 3.64 |  |  |
| 16 |  | 2.80 | 0.24 |  | 4.34 |  |  |
| 17 |  | 4.34 | 0.19 | 3.14 | 2.89 | 2.88 |  |
| 18 |  | 3.99 | 0.15 | 4.01 | 3.33 | 3.38 |  |
| 19 |  | 3.89 | 0.16 |  | 3.27 |  |  |
| 20 |  | 3.62 | 0.22 |  | 3.61 |  |  |
|  | mean |  |  | 3.48 | 3.39 | 3.08 | 3.81 |
|  | mean error |  |  | 0.15 | 0.08 | 0.09 | 0.18 |
| mean mean error |  |  |  | $\begin{aligned} & 3.58 \\ & 0.04 \end{aligned}$ |  |  |  |
|  |  |  |  |  |  |  |  |

brightness of many galaxy disks and this method is only once applied. In our investigated field there are six pairs of stars (near star - distant star) projected closely on the sky and for these stars it is very probable, that their radiation is crossing the same cloud. For these pair stars $(3,7),(7,6),(10,6),(13,9),(16,11)$ and $(19,17)$ the extinction $E(\lambda-V) / E(B-V)$ versus $1 / \lambda$ was calculated and the result of this cal-
culation indicated no difference for $V, B, U, 0.33,0.25$ and 0.22 bands. For 0.18 and 0.155 bands the differences are very clear - Figure 5. Adopting for every star (see Table 1) the value of $R$ for instance that calculated by method 1 and the value of extinction (see Table 3) the fundamental physical and chemical parameters of interstellar medium: the minimal $a$ and maximal $a+d a$ diameter of grain dust the parameter $p$, the ratio of number $n(S i) / n(C)$ may be derived on the basis of the work by Szczerba (1990).
It was calculated for 3 parameters model of dust grains assuming in this investigated field in Cassiopeia the mean value of $R=3.33$, the mean extinction calculated from all stars established in Table 3 - see Figure 6 and Table 5. The detailed analysis of the physical and chemical composition of interstellar dust grains will be made in the next paper.

Table 5.
The proposed composition of grains dust in investigated field in Cassiopeia calculated on the basis of revision model of $M R N$ (see Szczerba 1990)

$$
\begin{gathered}
R=3.33 \\
N(H) / E(B-V)=5.8000 D+21 \\
a=5.00000 D-03 \\
d a=2.71677 D-01 \\
p=3.32130 D+00 \\
A(C)=6.05531 D-25 \\
A(S i)=8.00404 D-25 \\
A(C) / E(B-V)=2.4401 D+00 \\
A(S i) / E(B-V)=1.6999 D+00
\end{gathered}
$$

$$
\text { Number of } C \text { grains per } H \text { atom }=1.1039 D-10
$$

$$
\text { Number of } S i \text { grains per } H \text { atom }=1.4592 D-10
$$

$$
\text { Number of Si to } C=1.3218 D+00
$$

$$
\text { Mass of } C \text { grains per mass of } H=3.8044 D-03
$$

$$
\text { Mass of } S i \text { grains per mass of } H=7.3428 D-03
$$

$$
\text { Mass of Si to } C \text { grains }=1.9301 D+00
$$

$$
\text { Surface of } C \text { per } H \text { atom } \quad=\quad 1.8157 D-21
$$

$$
\text { Surface of } S i \text { per } H \text { atom }=2.4001 D-21
$$

$$
\text { Total surface of grains per } H=4.2158 D-21
$$

Depletions in (\%)
revision model of MRN cosmic $n(i) / n(H)$

| $C$ | 75.48 | $4.200 D-04$ |
| :---: | ---: | ---: |
| $S i$ | 112.21 | $3.800 D-05$ |
| Mg | 106.60 | $4.000 \mathrm{D}-05$ |
| Fe | 125.41 | $3.400 \mathrm{D}-05$ |
| O | 20.98 | $8.130 \mathrm{D}-04$ |

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## Figure captions

## Figure 1.

The extinction curves derived from the $T D-1$ spectrum of $H D 144217$ ( $\beta^{1} S c o$ ) with the aid of $B 0 \mathrm{~V}$ artificial standard (dotted line) and $B 1 V$ artificial standard (solid line). Note the presence of the remnants of the strong CIV spectral feature in the form of emission or absorption "spectral lines" absent in the mean curve (open circles).

## Figure 2.

The extinction curve of $\beta^{1} S c o$ calculated with the aid of $B 3 V$ standard. Open circles - the same as in Figure 1. Note the "blueing" between $2740 \AA$ and $2540 \AA$, the change of $2200 \AA$ bump depth and the growing intensity of remnant $C I V$ feature - the result of spectral mismatch. The bump position remains unchanged.

## Figure 3.

Comparison of the $R$-dependent relation (equation 1) derived from extinction data for the three stars with different $R$ values.

## Figure 4.

The relation between $\Delta A_{V}=\Delta m_{p v}$ and $\Delta A_{B}=\Delta m_{p g}$ in 162 smaller ( $20^{\prime} \times 20^{\prime}$ ) fields in Cassiopeia. The slope $a=\Delta A_{V} / \Delta A_{B}, R=a /(1-a)$.

Figure 5.
The run of $E(\lambda-V) / E(B-V)$ versus $1 / \lambda$ for 6 near (dots circles-the mean distant amount to $1850 \pm 150 \mathrm{pc}$ ) and for 6 distant stars (open circles-the mean distant amount to $2350 \pm 250 \mathrm{pc}$ ) situated in neighbouring on the sky. The differences $m_{1800}-m_{2200}$ and $m_{1550}-m_{1800}$ for these stars indicated to the difference in the composition of grains dust.

## Figure 6.

Comparison between the observational data of Savage and Mathis (1979) - filled circles with best fit (solid line) obtained for mean value of $R=3.33$ and for mean value of $E(\lambda-V) / E(B-V)$ for all stars established in Table 3 with the assumption that graphite and silicate grains can have different size distributions. The separate contributions of silikate (short-dashed) and graphite (long-dashed line) are also shown.





A (V)


6пI



