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CONTINUOUS APPROXIMATIONS AND APPROXIMATE SELECTIONS FOR MULTIFUNCTIONS WITH VALUES IN S-CONTRACTIBLE SPACES

The notion of convexity was generalized by many different means /see [14] /. Some of these notions are useful in certain questions of topology and mathematical analysis, see for example [3, 10, 12, 16 - 20 , 22 - 24] . The present paper contains some extensions of existing theorems concerning continuous single-valued approximations and approximate selections for convex - valued multifunctions [1, 2, 4-9, 13, 15 , 21] onto the case of multifunctions whose values are S- convex subsets of a suitable S-contractible space. For to make our arguments reasonable complete we will start with recalling some basic notions related to S-convexity already discussed in detail in papers [16-20] and [22-23] .

A set Y is S- linear if there is a map $S : Y \times [0,1] \times Y \to Y$ such that:

/1/ $S(a,0,b) = b$ and $S(a,1,b) = a$ for all $(a,b) \in Y \times Y$.

The pair (Y,S) is then a convex prestructure in the sens of Gudder-Schroek [10] . For any subset B of a S-linear set Y define:

/2/ $coS(B) := \bigcap \{D \subset Y : B \subset S(B \times [0,1] \times D) \subset D\}$,

For $B = \emptyset$ we have $coS\ \emptyset = \emptyset$. A map $coS : 2^Y \to 2^Y$ defined by /2/

is a convex prehull on Y , i.e. the following two conditions /3/ and /4/ are satisfied:

/3/ $B \subset coS(B)$ for any $B \subset Y$,

/4/ $B \subset D \Rightarrow coS(B) \subset coS(D)$ for any $B, D \subset Y$.

Thus the family:

/5/ $C := \{B \subset Y : B = coS\ B\} \subset 2^Y$

determined by the convex prehull /2/ is a generalized convexity on Y. This means that:

/6/ $Y \in C$ and

/7/ $\{B_j : j \in J\} \subset C \Rightarrow \bigcap_{j \in J} B_j \in C$.

The elements of /5/ are called S-convex subsets of (Y, S) and C is called S-convexity. Note that in general $coS \circ coS \neq coS$ /see example 2 on p.17 in [22] /.

If Y is in addition endowed with some topological structure T, then (Y,T,S) will be called S-contractible, if for each $a \in Y$

/8/ $S(a, \cdot, \cdot) : [0,1] \times Y \to Y$ is a homotopy joining the identity $S(a,0,\cdot) = id_Y$ with a constant map $S(a,1,\cdot) = const_a$. In other words for every $a \in Y$ the map $h_a : [0,1] \to C(Y, Y)$ defined by:

/8/ $[0,1] \ni t \mapsto h_a(t) \in C(Y,Y)$, where

/9/ $Y \ni b \mapsto h_a(t)(b) := S(a,t,b) \in Y$

is continuous. The space $C(Y,Y)$ of all continuous transformations of the space Y in /8/ is assumed to be equiped with the quasi-compact open topology.

An S- contracible space (Y,T,S) is of type I /cf. [17], df. 3 on p. 596/ if for any $y \in Y$ and any neighbourhood V of y there exists a neighbourhood N of y such that $\operatorname{coS}(N) \subset V$. A space (Y,T,S) is of type O if it is S-contractible and fory any $B \subset Y$ and any neighbourhood V of the closure of coS B there exists a neighbourhood N of B for which $\operatorname{coS}(N) \subset V$ /cf. [18], df. 2.8 on p. 784/.

Let us suppose that the topology T is metrisable by a distance function $d: Y \times Y \to R^{+}$. By $K(b,r) := \{y \in Y : d(b,y) < r\}$ we denote the open ball centered at $b \in Y$ and of radius $r > 0$. Similarly, for any subset $B \subset Y$ the sign $K(B,r)$ will denote the set:

/10/ $K(B,r) := \bigcup \{K(b,r) : b \in B\}$.

A metric S-contractible space (Y,d,S) is called to be uniformly of type O for balls if

/11/ $$\bigwedge_{\varepsilon > 0} \quad \bigvee_{r(\varepsilon) > 0} \quad \bigwedge_{B \subset Y} \operatorname{coS} K(B, r(\varepsilon)) \subset K(\operatorname{coS} B, \varepsilon)$$

nd Y is of type I for this S.

Observe that each convex subset Y of any linear normed space is uniformly of type O if we define:

/12/ $S(a,t,b) := t \cdot a + (1-t) \cdot b \in Y$.

Without loss of generality we can on the strength of /4/ always assume in /11/ that the following inequality holds:

/13/ $0 < r(\varepsilon) \leq \varepsilon$.

et (X,T_1) be another topological space and let us consider a ultifunction $F: X \to Y$, i.e. a function whose values are nonempty subsets of Y. F is called lower semicontinuous /briefly lsc/ at a point $x_0 \in X$ iff whenever W is an open set in Y with the property that $F(x_0) \cap W \neq \emptyset$, there exists a neighbourhood U of x_0 such that $F(x) \cap W \neq \emptyset$ for every $x \in U = U(x_0)$. F is calle almost lower semicontinuous /alsc/ at x_0 (see [8], df. 2.1 on p. 186) iff for each positive real number $\varepsilon > 0$, there exists a neighbourhood $U = U(x_0)$ of x_0 such that:

$$/14/ \quad \bigcap \left\{ K\left(F(x), \varepsilon\right) : x \in U(x_0) \right\} \neq \emptyset .$$

F is called lower semicontinuous /resp. almost lower semicontinuous/ if it is lsc /resp. alsc/ at each point x_0 of X.
A selector /resp. ε - approximate selector / for an F is a single valued function $f : X \to Y$ such that $f(x) \in F(x)$ / resp. $f(x) \in K\left(F(x), \varepsilon\right)$ for every x in X. Observe that every selector is an ε - approximate selector, but the converse is false in general. It is useful for comparison purposes to mention here the L.Pasicki analogue of celebrated continuous selection theorem of E. Michael:

PROPOSITION 0 /L.Pasicki/. Let X be a paracompact topological space and (Y, d, S) an S-contractible metric space uniformly of type 0 for balls. Suppose that $F: X \to Y$ is a multifunction with S-convex complete values /resp. S-convex values only/. If F is lower semicontinuous, then F admits a continuous selector /resp. a continuous ε - approximate selector for each $\varepsilon > 0$ / .
While lower semicontinuity of F with complete S-convex values is sufficient for the existence of a continuous selector, it is in general not necessary for F to admit either a continuous selector or even a continuous ε - approximate selector.

In case where Y is a normed linear space, Deutsch and Konderov ([8], thm. 2.4 on p. 187) have characterized almost lower semicontinuity and in the process showed that it is a necessary condition for the existence of a continuous selector. Our first proposition is an extension into S-contractible spaces of Deutsch-Kenderow theorem characterizing those multifunctions with S-convex images which have continuous ε - approximate selectors for every $\varepsilon > 0$.

PROPOSITION 1. Let (X,T_1) be a paracompact space and let (Y,d,S) be a metric S-contractible space uniformly of type O for balls. Let $F:X \to Y$ be a multifunction with S-convex values. Then F is almost lower semicontinuous if and only if for each $\varepsilon > 0$, F has continuous ε - approximate selector.

PROOF: Necessity: Suppose F is alsc and let an arbitrary positive number $\varepsilon > 0$ be given. Take $r(\varepsilon)$ satisfying /11/ and /13/. In compliance with /14/ for each $x_0 \in X$ there exists an open neighbourhood $U(x_0)$ of x_0 such that

$$/15/ \quad \bigcap \left\{ K\left(F(x), r(\varepsilon)\right) : x \in U(x_0) \right\} \neq \emptyset .$$

Since (X,T_1) was paracompact, the open cover $\{U(x) : x \in X\}$ of X has a locally finite refinement $\{V_j : j \in J\}$ where J is a set of indices. We can assume, without loss of generality, that the indexing set J is well ordered by some total order relation $\prec \subset J \times J$. For each $j \in J$ choose $x_j \in X$ such that $V_j \subset U(x_j)$. Using paracompactness, we can choose a partition of unity $\{p_j : j \in J\}$ subordonated to $\{V_j : j \in J\}$. That is, each function $p_j : X \to [0,1]$ is T_1 - continuous,

/16/ $$\bigwedge_{x \in X} \sum_{j \in J} p_j(x) \equiv 1 \quad \text{and} \quad p_j(X \setminus V_j) = \{0\}.$$

For each $x \in X$ define:

/17/ $$J(x) := \{ j \in J : p_j(x) \neq 0 \} =: \{ j_1, j_2, \ldots, j_n \}$$

where $n = n(x)$ is dependent of x and $j_1 \prec j_2 \prec \ldots \prec j_n$. Put:

/18/ $$c_k(x) := p_k(x) \,/\, \max \{ p_j(x) : j \in J(x) \} ; \quad k \in J(x).$$

Obviously each $c_k : X \to [0,1]$ is continuous. For each $j \in J$ let us select $y_j \in \bigcap \{ K(F(x), r(\varepsilon)) : x \in V_j \}$ and define $f : X \to Y$ by a formula:

/19/ $$f(x) := S(y_{j_1}, c_{j_1}(x), S(y_{j_2}, c_{j_2}(x),$$

$$S(\ldots, S(y_{j_n}, c_{j_n}(x), y) \ldots) \in Y$$

where y is an arbitrary fixed element of the image $F(X) \subset Y$.
It is easily seen that there always exists an $k \in J(x)$ such that $c_k(x) = 1$, then $S(y_k, c_k(x), y) = y_k$ for $y \in Y$.
Then our definition /19/ is correct and $f(x)$ is independent of the choose of y.
Given any $x_0 \in X$, there is a neighbourhood $O(x_0)$ which intersects only finitely many of the V_j so $x_0 \in V_j$ for only a finite set of indices $J(x_0) \subset J$. We have:

/20/ $$\bigwedge_{x \in O(x_0)} c_j(x) = 0 \Leftrightarrow j \in J \setminus J(x_0).$$

Consequently for all $x \in O(x_0)$ we essentially take in /19/ those y_j, for which $j \in J(x_0)$. Observe that the function:

/21/ $$O(x_0) \ni x \mapsto g_n(x) := S(y_{j_n}, c_{j_n}(x), y) \in Y, \quad n = n(x_0)$$

is continuous on $O(x_o)$. For $i = 1,2, \ldots, n-1$ let us define recursively:

/22/ $O(x_o) \ni x \mapsto g_{n-i}(x) := S(y_{j_{n-i}}, c_{j_{n-i}}(x), g_{n-i+1}(x)) \in Y$.

Since $y_{j_{n-i}}$ are constant on $O(x_o)$ and $S(y_{j_{n-i}}, \ldots) : [0,1] \times Y \to Y$ is jointly continuous as a homotopy, we infer that each g_{n-i} is continuous on $O(x_o)$ being a superposition of continuous maps. Thus $f \mid O(x_o) = g_1$ is continuous on $O(x_o)$. Since $\{O(x_o) : x_o \in X\}$ is an open covering of X, we infer that /19/ is continuous on X. Observe that:

/23/ $\bigwedge_{x \in X} \quad f(x) \in \operatorname{coS} K(F(x), r(\varepsilon))$.

In fact, choose any subset D belonging to the family under the sign of intersection in formula /2/, where $B := K(F(x), r(\varepsilon))$. Observe that for $i = 1,2,\ldots, n-1$ we have recursively:

/24/ $g_{n-i}(x) \in S(B \times [0,1] \times D) \subset D$

for a function g_{n-i} defined by /22/, because of $g_{n-i+1}(x) \in D$ and $y_{j_{n-i}} \in B$. By $S(y_k, c_k(x), y) = y_k$ for some $k \in J(x)$ the choice of $y \in F(X)$ is unessential, even if $y \notin D$. Since D was arbitrary, this yields:

/25/ $g_1(x) = f(x) \in \operatorname{coS} B = \operatorname{coS} K(F(x), r(\varepsilon))$.

Bearing in mind that $F(x) = \operatorname{coS} F(x)$, by /11/ we obtain:

/26/ $f(x) \in \operatorname{coS} K(F(x), r(\varepsilon)) \subset K(\operatorname{coS} F(x), \varepsilon) = K(F(x), \varepsilon)$.

Thus:

/27/ $\operatorname{dist}(f(x), F(x)) := \inf\{d(f(x), y) : y \in F(x)\} < \varepsilon$

and f is a desired continuous ε- approximate selector for our multifunction F.

Sufficiency: Fix $\varepsilon > 0$ and $x_0 \in X$. Assume that for each $\eta > 0$ there is an $f_\eta \in C(X,Y)$ such that $f_\eta(x) \in K(F(x), \eta)$. Take $f := f_\eta$ for $\eta = \varepsilon/2$ and choose a neighbourhood $U(x_0)$ of x_0 such that $d(f(x_0), f(x)) < \varepsilon/2$ for all $x \in U(x_0)$. Such $U(x_0)$ exists since f was continuous. Hence

$$/28/ \quad \bigwedge_{x \in U(x_0)} \quad f(x_0) \in K(f(x), \varepsilon/2) \subset K(F(x), \varepsilon) .$$

In fact, if $y \in F(x)$ is such that $d(f(x), y) < \varepsilon/2$ then by the triangle inequality we have:

$$/29/ \quad d(f(x_0), y) \leq d(f(x_0), f(x)) + d(f(x), y) < \varepsilon/2 + \varepsilon/2 = \varepsilon$$

so that $\operatorname{dist}(f(x_0), F(x)) < \varepsilon$. Thus /14/ holds and F is alsc at x_0. Since $x_0 \in X$ was arbitrary, F is alsc as required and the proof of Proposition 1 is completed. ◻

At the present let us suppose that the topology T_1 on X is metrizable by a distance function d_1. For computational simplicity assume the Cartesian product X x Y to be endowed with the box metric d_2:

$$/30/ \quad d_2((x_1, y_1), (x_2, y_2)) = \max\{d_1(x_1, x_2), d(y_1, y_2)\}$$

A function $f : X \to Y$ is called ε- approximation for multifunction $F: X \to Y$ if:

$$/31/ \quad H^*(\mathrm{Gr}\, f, \mathrm{Gr}\, F) \leq \varepsilon ,$$

where the separation H^* is defined on X x Y by formula:

$$/32/ \quad H^*(M,N) := \sup_{n \in N} \inf_{m \in M} d_2(m, n) ; \quad M,N \subset X \times Y$$

and the graph of F is defined as usualy by:

$$/33/ \quad \mathrm{Gr}\, F := \{(x,y) \in X \times Y : y \in F(x)\} .$$

Each ε- approximate selector for F is simultaneously its ε-appr ximation, but the converse is not true in general. Consider, as a example, the multifunction $F: R \to R$ given by the formula:

$$/34/ \qquad F(x) := \begin{cases} \{\operatorname{sgn} x\} & \text{for } x \neq 0 \\ [-1, 1] & \text{for } x = 0 \end{cases} .$$

It is impossible to inscribe into the graph of /34/ a single-valued continuous function, i.e. there is no continuous selector for F.

Even more, it is also impossible to find a sequence f_n of continuous single-valued functions such that:

$$/35/ \quad \text{dist} \left(f_n(x) , F(x) \right) \rightrightarrows 0$$

uniformly /or almost uniformly/ on R as n tends to infinity.

In /35/ the sign dist is defined by formula /27/. This example /34/ shows that in the theory of multifunctions neither the simple inscription concept nor the traditional approximation principle may lead to general and satisfactory result. One feels that here some more sophisticated principle is needed. It is easily seen that it is possible to find a sequence f_n of continuous single valued functions such that the sequence $\text{Gr } f_n \subset X \times Y$ of their graphs converges to the graph /33/ of the multifunction F, i, e. $H^*(\text{Gr } f_n, \text{Gr } F)$ tends to zero as n tends to infinity /cf.[4-7, 13, 15, 21]/.

Observe that /34/ fails to be alsc at $x_0 = 0$ and thus, in compliance with Proposition 1 admits no continuous ε- approximate selector for sufficiently small numbers $\varepsilon > 0$. Following [5], df. 1.7 on p. 13 , a multifunction $F: X \to Y$ is called weakly h^* - upper semicontinuous /briefly weakly h^* - usc/ at $x_0 \in X$ if

$$/36/ \quad \bigwedge_{\varepsilon>0} \bigwedge_{\eta>0} \bigvee_{0<\delta\leq\eta} \bigvee_{x_1\in K(x_0,\delta)} \bigwedge_{x\in K(x_0,\delta)} h^*\big(F(x),\ F(x_1)\big) < \varepsilon$$

where similarly as in /32/ the separation is defined by:

$$/37/ \quad h^*(B,D) = \sup\left\{ \operatorname{dist}\,(b,\ D) : \quad b \in B \right\}$$

and the sign dist is explained by /27/. Observe that in general $h^*(B,D)$ differs from $h^*(D,B)$. A multifunction $F:X \to Y$ is called weakly h^* - upper semicontinuous if it is weakly h^* - usc at each point $x_0 \in X$. $F:X \to Y$ is called h^* - usc iff:

$$/38/ \quad \bigwedge_{x_0\in X} \bigwedge_{\varepsilon>0} \bigvee_{\delta>0} \bigwedge_{x\in K(x_0,\delta)} h^*\big(F(x)\ ,\ F(x_0)\big) < \varepsilon \ .$$

If $x_1 = x_0$ in /36/ the definition of weakly h^* -upper semicontinuity reduces to that of an h^* -upper semicontinuity. While each h^* -usc multifunction is weakly h^* - usc, the converse is not true in general. In the abbreviation " h^* -usc ", h^* is written to emphasize the role of the Pompeiu-Hausdorff /generalized/ separation /37/. If card $F(x) = 1$ for all $x \in X$, i.e. $F(x) = \{f\ (x)\}$ is single valued then F is h^* -usc /lsc, alsc, weakly h^* -usc / if and only if f is continuous. Following [13], p. 72 define:

$$/39/ \quad D\big(x,\varepsilon\big) := \Big\{ 0<\delta\leq\varepsilon : \bigvee_{x_1\in K(x,\delta)} \bigwedge_{x_2\in K(x,\delta)} h^*\big(F(x_2),\ F(x_1)\big)<\varepsilon \Big\} =$$

$$= \Big\{ 0<\delta\leq\varepsilon \quad : \bigvee_{x_1\in K(x,\delta)} F\big(K\ (x,\delta)\big) \subset K\big(F\ (x_1)\ \ ,\varepsilon\big) \Big\}$$

where for a subset $A \subset X$ we define the image as:

$$/40/ \quad F(A) := \bigcup \left\{ F(a)\ :\ a \in A \right\} \subset Y \ .$$

Observe that $D\ (x,\varepsilon)$ is certainly nonempty if F is weakly h^* -usc and if $\varepsilon > 0$. The function $\delta : X \times R^+ \to R^+$ defined by:

/41/ $\delta(x, \varepsilon) := \sup (D(x, \varepsilon))$

is called in [13] the modulus of upper semicontinuity of the multifunction F. If $F : X \to Y$ is weakly h^*-usc then the modulus of upper semicontinuity /41/ is positive and lower semicontinuous with respect to the first variable x / see [5], lemma 3.2 on p. 20, cf. also a lemma on p. 72 in [13]/.

PROPOSITION 2. Let (X, d_1) be a metric space and (Y, d, S) a metric S-contractible space uniformly of type 0 for balls. Let $F: X \to Y$ be a weakly h^*-upper semicontinuous multifunction with S-convex values. Then for every $\varepsilon > 0$ there exists a continuous ε-approximation for F, i.e. a single valued mapping $f \in C(X,Y)$ such that the inequality /31/ is fulfilled. Moreover:

/42/ $f(X) := \{f(x) : x \in X\} \subset \operatorname{coS} F(X) := \operatorname{coS}\left(\bigcup_{x \in X} F(x)\right) \subset Y.$

PROOF: For a given $\varepsilon > 0$, we define a multifunction $G: X \to Y$ by putting:

/43/ $G(x) := F(K(x, \delta(x, r(\varepsilon/2))))$

for every $x \in X$, where $r = r(\varepsilon/2)$ is taken from /11/. We claim that for every y in Y the fiber:

/44/ $G^{-1}(y) := \{x \in X : y \in F(x)\}$

is open in X, i.e. that multifunction /43/ is strongly lower semicontinuous. Indeed, if x_0 belongs to the fiber /44/ then:

/45/ $y \in G(x_0) = F(K(x_0, \delta(x_0, r)))$, $r = r(\varepsilon/2)$.

This means that $y \in F(x_1)$ for certain x_1 belonging to the ball $K(x_0, \delta(x_0, r(\varepsilon/2)))$. By the lower semicontinuity of the function $X \ni x \mapsto \delta(x,r) \in R^+$, there exists an $\eta > 0$ such that for all $x \in K(x_0, \eta)$ we have $x_1 \in K(x, \delta(x,r))$, $r = r(\varepsilon/2)$, in accordance with /39/, /41/ which means that for all such points x we have $y \in G(x)$. Since $x_0 \in G^{-1}(y)$ was arbitrary, this means that the fiber /44/ is open in X.

The family $\{G^{-1}(y) : y \in Y\}$ is an open covering of the space X. Since every metric space X is paracompact, there exists a locally finite refinement $\{V_j : j \in J\}$ of this covering. Now, let $\{p_j : j \in J\}$ be a partition of unity subordonated to this refinement, so that /16/ holds. Choose for every $j \in J$ a point $y_j \in Y$ such that $V_j \subset G^{-1}(y_j)$ and define $f: X \to Y$ by a formula /19/ where c_j and $J(x)$ are defined by /18/ and /17/ respectively. We can prove in exactly the same manner as in the proof of Proposition 1 that the function f is continuous on X. For an arbitrary x in X, $f(x)$ is an S-convex combination of a finite number of y_j such that:

/46/ $y_j \in F(x_j) \subset G(x)$, $x_j \in K(x, \delta(x, r))$.

Fix now x arbitrary. By the definition /43/ of G, there exists a point x_1 such that:

/47/ $d_1(x, x_1) < \delta(x, r(\varepsilon/2))$ and

/48/ $G(x) = F(K(x, \delta(x,r))) \subset K(F(x_1), r)$ where $r = r(\varepsilon/2)$

and /39/ is utilized. Since $F(x_1) = \cos F(x_1)$ we have by /46/, /48/ and /11/ that:

/49/ $f(x) \in \cos G(x) \subset \cos K(F(x_1), r) \subset K(\cos F(x_1), \varepsilon/2) =$
$= K(F(x_1), \varepsilon/2)$.

Thus, arguing similarly as in /29/ we obtain:

/50/ $\operatorname{dist}\left(f(x), F(x_1)\right) < \varepsilon/2$.

Subsequently we have by virtue of the triangle inequality:

$$/51/\ \operatorname{Dist}\left((x, f(x)), \operatorname{Gr} F\right) := \inf\left\{d_2\left((x, f(x)), (u,v)\right) : (u,v) \in \operatorname{Gr} F\right\} \leq d_2\left((x, f(x)), (x_1, f(x))\right) + \operatorname{Dist}\left((x_1, f(x)), \{x_1\} \times F(x_1)\right) \leq d_2\left((x, f(x)), (x_1, f(x))\right) + \operatorname{dist}\left(f(x), F(x_1)\right) \leq \delta(x,r) + \varepsilon/2 \leq r + \varepsilon/2 \leq \varepsilon/2 + \varepsilon/2 = \varepsilon ,$$

where the inequalities /47/, /50/, /39/ and /13/ are adequately taken into consideration. Since x was arbitrary the proof of /31/ is completed. The inclusion /42/ follows from /46/ in a manner appearing in the proof of formula /26/. That ends the proof.

PROPOSITION 3. Let X and Y be the same as in Proposition 2 and let $F: X \to Y$ be an upper semicontinuous multifunction with closed values. If $f_{\varepsilon_n} : X \to Y$ is a sequence of ε_n - approximations for F, where ε_n create a sequence tending to zero as n tends to infinity, then for every converging sequence x_n of points of the domaine X satisfying the equality $\lim_{n\to\infty} f_{\varepsilon_n}(x_n) = y_0$ we have $y_0 \in F\left(\operatorname{Lim}_{n\to\infty} x_n\right)$.

PROOF: This follows immediately from Theorem 1.5.3 announced in a survey [25] and from our Proposition 2.

For more informations about continuous approximations for multifunctions the reader is refered to papers [1,2, 4-9, 13,15,21,26,27] and to soviet works of Y.G.Borisovich, A.D.Myshkis, B.D.Gelman, Y.E.Glicklich and others, carefuly surveyed in [25] . The role of approximations in the theory of multifunctions was emphasized in

[26]. The author wishes to express his thanks to Janina Ewert for her critical remarks.

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CIĄGŁE APROKSYMACJE I APROKSYMATYWNE SELEKTORY DLA MULTIFUNKCJI O WARTOŚCIACH W PRZESTRZENIACH S - ŚCIĄGALNYCH

Streszczenie

W pracy sformułowano warunki przy których multifunkcja przyjmująca S-wypukłe wartości we wprowadzonej przez L.Pasickiego przestrzeni S-ściągalnej odpowiedniego typu posiada dla każdej $\varepsilon > 0$ ciągły ε-aproksymatywny selektor oraz odpowiednio ciągłą jednowartościową ε-aproksymację. Uzyskane wyniki rozszerzają zakres stosowalności twierdzeń znanych w przypadku multifunkcji przyjmujących wartości wypukłe w lokalnie wypukłych przestrzeniach liniowo-metrycznych wskazując jednocześnie na nieco inne zastosowania S-wypukłości w teorii multifunkcji niż w pracach [17-20] i [22-23].