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NUMERICAL STUDY OF LARGE SCALE TRUSS CONSTRUCTION WITH STRUCTURAL UNCERTAINTIES

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Abstarct: *The paper deals with efficiency and usability problem for the chosen solution methods for mechanical systems with structural uncertainties. They are significantly influencing the analysis results and the analysis itself. An application of the chosen approaches will be presented – the first one, a simple combination of only inf-values or only sup-values; the second one presents full combination of all inf-sup values; the third one uses the optimizing process as a tool for finding out an inf-sup solution and last one is Monte Carlo method as a comparison tool.*

Keywords: *Uncertain parameters, uncertainties and errors, Monte Carlo method, interval arithmetic, optimization, MATLAB*

1. INTRODUCTION

In the last years there has been an increased interest in the modeling and analysis of engineering systems under uncertainties. To obtain reliable results for the solution of engineering problems, exact values for the parameters of the model equations should be available. Really, however, those values can often not be provided, and the models usually exhibit a rather high degree of uncertainty.

Computational mechanics, for example, entails uncertainties in geometry, material and load parameters as well as in the model itself and in the analysis procedure too. For that reason, the responses, such as displacements, stresses, resonant frequency, or other dynamic characteristics, will usually exhibit any degree of uncertainty. It means that the obtained result using one specific value as a most significant value for uncertain parameter cannot be considered to be representative for the whole spectrum of possible results.

Uncertain parameters appear mostly as random variables and thus are described in the terms of stochastic approach. But without the knowledge of the probability density and the nature of distribution we are forced to use another approach, which could describe the parameters with the mentioned restrains and at the same time contain sufficient information about the character of the uncertainty.

Alternately to the use of probability methods we can use imprecise probabilities and the possibility theory [1], which involves the theory of interval numbers, fuzzy numbers and fuzzy sets [2, 3, 4, 9]. Without the information of the relevance of the data on the interval, we cannot use the fuzzy approach, but we are still able to use the interval approach to describe the uncertain parameters which are considered as unknown but bounded with lower and upper bounds.

Our short study proposes algorithms for modal and spectral interval computations of FE models and their efficiency analysis in view of the input uncertainty degree (20%, 50%) [5, 9].

2. UNCERTAINTIES AND ERRORS IN FINITE ELEMENT ANALYSIS

The accuracy of FEA is affected by errors and uncertainties, which may be related to the numerical tool itself or to the physics of the problem. The possible sources of uncertainties and errors in FEA include model uncertainty, discretization error, parameter uncertainty and rounding error [10]. The definitions of these uncertainty and errors are summarized and presented below.

Model uncertainty

The mathematical model in FEA represents the physical system being analyzed. The actual problem is simplified and idealized, and is described by an accepted mathematical formulation such as the theory of elasticity, or thin-plate theory and so on [6, 7, 8]. The uncertainty about how well the mathematical model represents the true behavior of the real physical system is termed model uncertainty.

Typical model uncertainties in FEA are:

- the idealization of the boundary conditions,
- the use of plane model rather than three-dimensional model,
- the use of linear model rather than nonlinear model,
- the use of time-independent model rather than dynamic model.

Discretization terror

The esTablelished mathematical model is represented by an FE discretization. This involves selecting a mesh and elements. The computed solution of the FE model is in general only an approximation of the exact solution of the mathematical model, and the discrepancy is called discretization error. FEA solution is influenced by the factors, such as the number of elements used, the nature of element shape functions, integration rules used, and other formulation details of particular elements.

Parameter uncertainty

Parameter uncertainty arises because the precise data needed for the analysis are not available. This type of uncertainty is sometimes called parametric or data uncertainty. In FEA, the parameter uncertainty may exist in the geometrical, material or loading data. Parameter uncertainty may result from a lack of knowledge, an inherent variability in the parameters, or both.

Rounding error

FEA solution is limited in accuracy by the finite precision of computer arithmetic. When arithmetic operations are performed on floating point numbers, the exact result will not, in general, be represenTablele as a floating point number. The exact result will be rounded to the nearest floating point number, and this loss of information is referred to as rounding error. A more fundamental approach, however, is to use interval arithmetic. Interval arithmetic can rigorously bound the rounding error.

3. COMPUTATIONAL METHODS FOR INTERVAL ANALYSIS

If we want to use interval arithmetic approach, an uncertain number is represented by an interval of real numbers [2, 3]. The interval numbers derived from the experimental data or expert knowledge can then take into account the uncertainties in the model parameters, model inputs etc. Complete information about the uncertainties in the model may be included by this technique and one can demonstrate how these uncertainties are processed by the calculation procedure in MATLAB.

During the solving of the particular tasks using the interval arithmetic application on the solution of numerical mathematics and mechanical problems, the problem known as the overestimate effect is encountered. Its elimination is possible only in the case of meeting the specific assumptions, mainly related to the time efficiency of the computing procedures.

Considering uncertain parameters in interval form, some solution approaches already used or proposed by the authors are analyzed [5, 9]. The goal is to present algorithm description and comparison study of the following numerical methods:

Monte Carlo method (MC) also as a comparison tool,

a simple combination of only inf-values or only sup-values (COM1),

a full combination of all inf-sup values (COM2),

a method which uses an optimization process as a tool for finding out a inf-sup solutions (OPT).

Monte Carlo method (**MC**) is a time consuming but reliable solution. Various combinations of the uncertain parameter deterministic values are generated and after the subsequent solution in the deterministic sense we obtain a complete set of results processed in an appropriate manner. Infimum and supremum calculation is following

$$\inf(F) = \min \text{ of all results } F(p_i), \text{ where } i = 1,\ldots,m \text{ and } m \approx 5000 \div 100000, \quad (1)$$
$$\sup(F) = \max \text{ of all results } F(p_i), \text{ where } i = 1,\ldots,m \text{ and } m \approx 5000 \div 100000$$

Solution evaluation in marginal values of interval parameters (**COM1**) has its physical meaning for many engineering problems. We consider an approach where the extreme output values are obtained by the application of the extreme parameter values on input. That means that the infimum or supremum is obtained using the deterministic analysis for infimum or supremum of input uncertain parameters. Inf-sup calculation is

$$\inf(F) = \min \text{ of } \left[F(\underline{p}), F(\overline{p})\right] , \qquad (2)$$
$$\sup(F) = \max \text{ of } \left[F(\underline{p}), F(\overline{p})\right]$$

Solution evaluation for all marginal values of interval parameters (**COM2**) which is also based on the set of the deterministic analyses appears as the more suiTablele one. The marginal interval parameter values are considered again but the inf and sup are also combined. The method provides satisfying results and can be marked as reliable, even if there is still a doubt about the existence of the extreme solution for the uncertain parameter inner values.

A solution for two interval numbers $p_1 = \langle a_1 \; b_1 \rangle$ and $p_2 = \langle a_2 \; b_2 \rangle$ may be found in the following computational way

$$\inf(F) = \min \text{of } F \left[(a_1 \; a_2), \; F(a_1 \; b_2), \; F(b_1 \; a_2), \; F(b_1 \; b_2)\right] ,$$
$$\sup(F) = \max \text{of } F \left[(a_1 \; a_2), \; F(a_1 \; b_2), \; F(b_1 \; a_2), \; F(b_1 \; b_2)\right] .$$
(3)

The method of the inf and sup solution using the optimization techniques (**OPT**) is proposed by the authors as an alternative to the first and to the third method. It should eliminate a big amount of analyses in the first method and also eliminates the problem with the possibility of the inf and sup existence inside of the interval parameters for the deterministic values. Computational process for two interval numbers p_1 and p_2 may be found as follows

$$\inf(F) = F(\mathbf{p}_{OPT}) \text{ , i.e. find } \mathbf{p}_{OPT} \text{ so that } F(\mathbf{p}_{OPT}) \to \min$$
$$\sup(F) = F(\mathbf{p}_{OPT}) \text{ , i.e. find } \mathbf{p}_{OPT} \text{ so that } F(\mathbf{p}_{OPT}) \to \max$$
(4)

4 SOLVING OF TRUSS STRUCTURE WITH INTERVAL PARAMETERS

Considering different uncertain parameters the numerical interval stress-strain study of a three-dimensional truss structure was performed. The geometry of the structure is presented on Figure. 1. The truss structure was loaded by forces F in all upper nodes of the structure. The truss structure consists of 70 nodes and 257 bars.

The certain and uncertain model parameters are defined as follows:

element mass density	$\rho = 7800 \text{ kg} \cdot \text{m}^{-3}$,
Young's modulus	$E = 2.1 \cdot 10^{11}$ Pa ,
cross-section areas	$A = 0.0015 \text{ m}^2$,
force	$F = 1000$ N .

The force, cross-section area and Young's modulus were used as the uncertain parameters. The uncertainty degree was implemented for values of 20% and 50%.

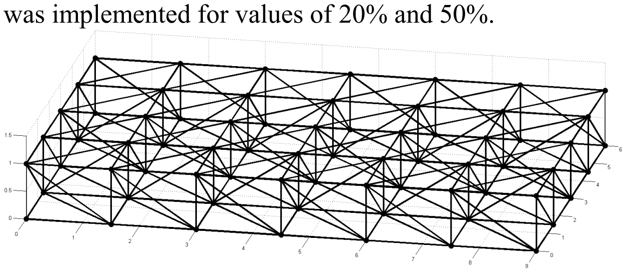

Figure. 1 Analyzed truss structure, dimensions in [m]

The purpose of this study will be to compare the efficiency and exactness of the proposed methods MC, COM1, COM2 and OPT. The results of the MC analysis will be considered as reference values and will be used for the construction of the solution map. In the case of MC method, 10000 random inputs were generated; they were evaluated and properly processed to inf-sup solutions.

The maximal and minimal inf-sup stress values are summarized in Table. 1 and maximal displacement shows Table. 2.

Table. 1 Stress inf/sup results for the chosen bars [Pa]

Bar No.		Stress [Pa]	
		COM1	COM2
200	inf	**2446582,18**	**2304062,83**
	sup	**3117261,47**	**3310081,77**
206	inf	**-4833204,26**	**-5132165,35**
	sup	**-3793339,61**	**-3572368,37**
Bar No.		Stress [Pa]	
		OPT	MMC
200	inf	**2304062,83**	**2546957,37**
	sup	**3310081,77**	**3044004,29**
206	inf	**-5132165,35**	**-4719621,57**
	sup	**-3572368,37**	**-3948967,81**

Table. 2 **Displacement inf/sup result for the chosen node [m]**

Node No.		Displacement [m]	
		COM1	COM2
39	inf	**0,000291311**	**0,000224461**
	sup	**0,000303683**	**0,000394127**
Node No.		Displacement [m]	
		OPT	MMC
39	inf	**0,000224461**	**0,000130511**
	sup	**0,000322467**	**0,000170684**

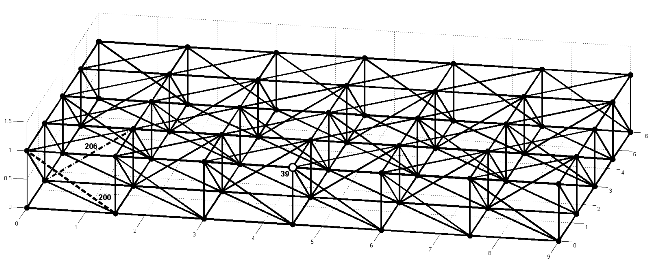

Figure. 2 **The maximal (bar No. 200), minimal (bar No. 206) stress values and maximal displacement (node No. 39)**

Mapping of the generated input data by MC method for bars with max and min stresses and for node with max displacement is shown on Figs. 3–5. Stress solution on the bar No. 206 for various uncertainties is shown on Figs. 6–7. Displacement solution on the node No. 39 for various uncertainties is shown on Figs. 8–9.

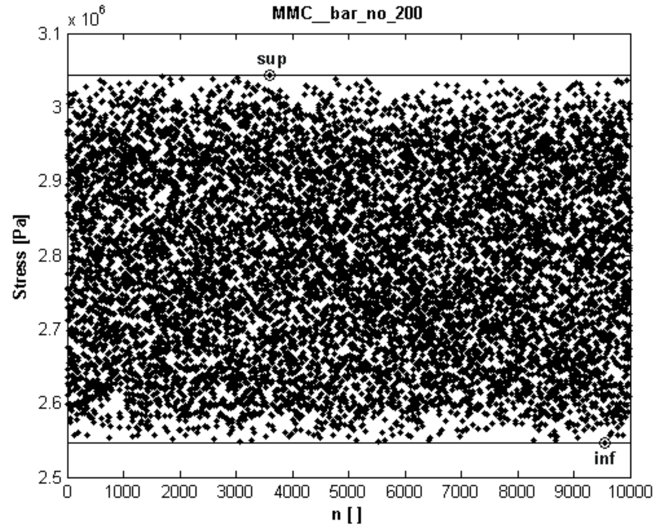

Figure. 3 **Mapping of the generated input data for bar No. 200**

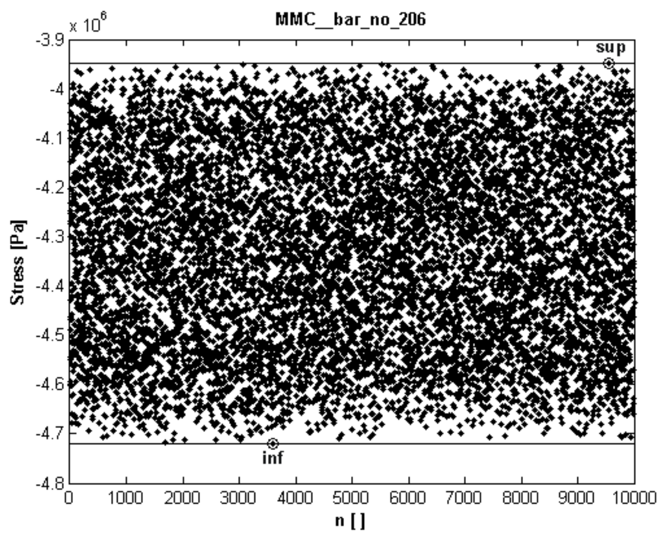

Figure. 4 **Mapping of the generated input data for bar No. 206**

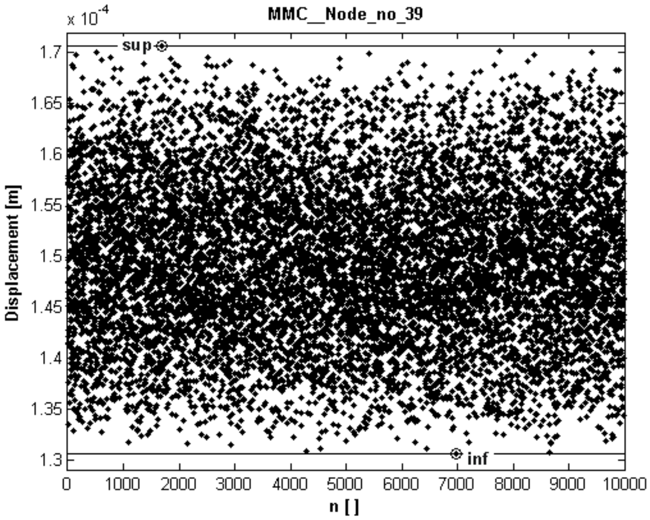

Figure. 5 **Mapping of the generated input data for node No. 39**

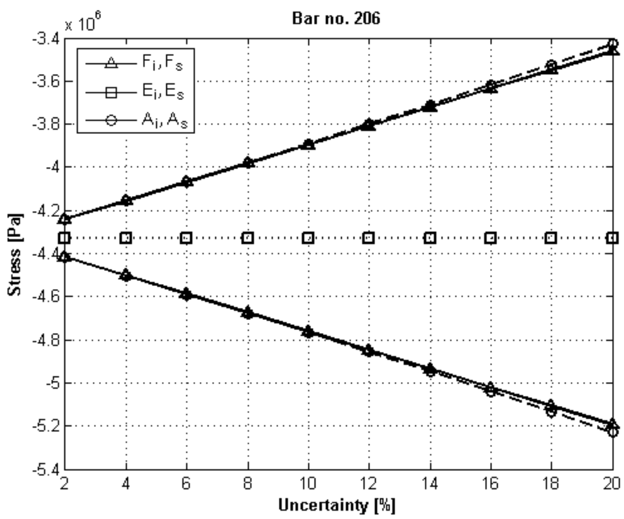

Figure. 6 **Stress solution on bar No. 206 (max uncertainty 20%)**

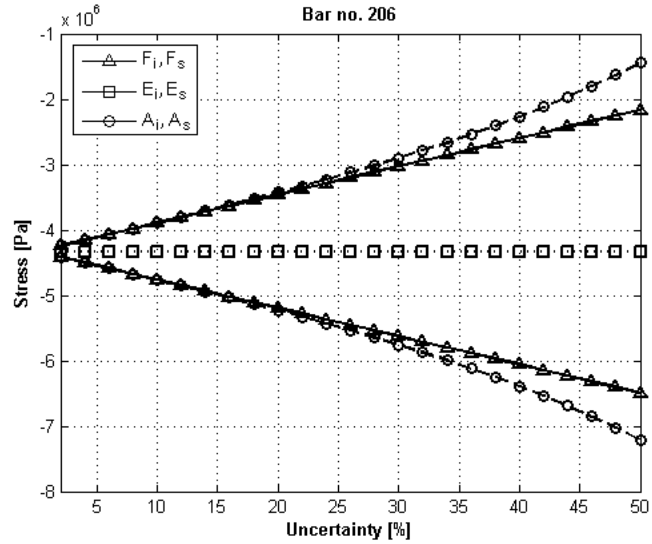

Figure. 7 **Stress solution on bar No. 206 (max uncertainty 50%)**

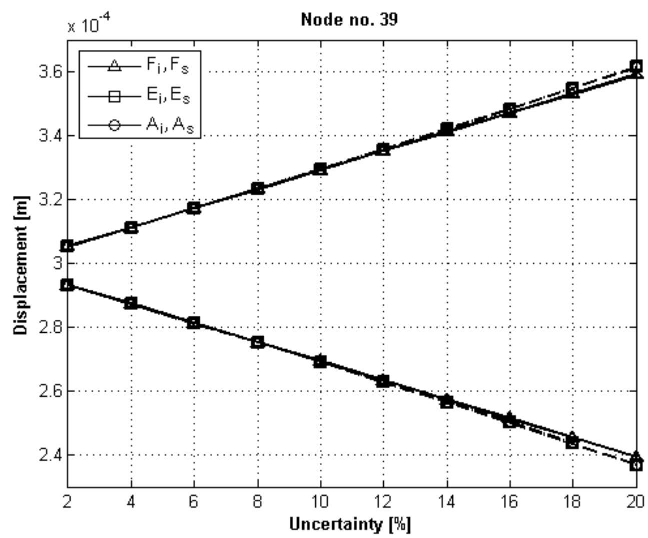

Figure. 8 **Displacement solution on node No. 39 (max uncertainty 20%)**

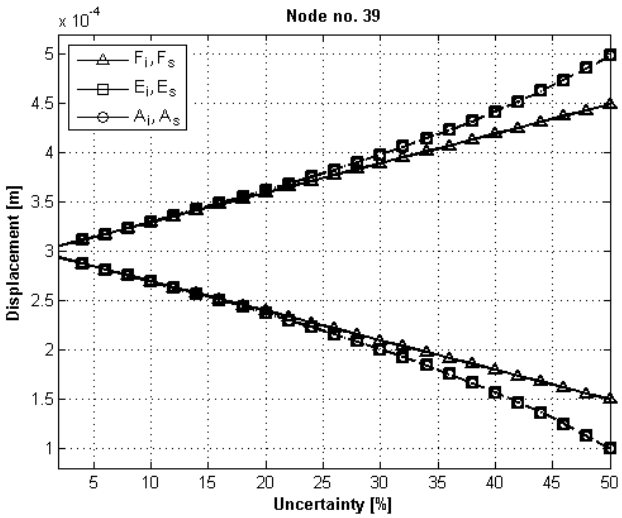

Figure. 9 **Displacement solution on node No. 39 (max uncertainty 50%)**

5. CONCLUSION

The paper presents methods and their applications in an interval structural analysis. The use of the interval arithmetic provides a new possibility of the quality and reliability appraisal of analyzed objects. It shows the stress-strain solution efficiency for solving problems including uncertain parameters with a various width of the interval.

The analyses results can be summarized as follows:

- COM2 method provides decent results, but it is limited due to the exponential growth of the analyses number for complicated problems,
- OPT method provides good results and is suiTablele for complicated problems because it does not need so many analyses as in the cases of the MC or COM2 methods,
- the cross-section area as uncertain parameter has the biggest influence on stress solution,
- the cross-section area and Young's modulus as uncertain parameters have the biggest influence on displacement solution.

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